Spillovers in the Foreign Exchange Market: A study of volatility and returns in emerging market currencies

Minor Dissertation[1][2]
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Abstract

This paper provides a rigorous investigation of spillover effects in exchange rate returns and volatility. It considers the construction of a spillover index for advanced and emerging market currencies including the South African rand. The results suggest that the spillover index of exchange rate returns have increased steadily over time and that it exhibits moderate reactions to economic events. In contrast, spillovers in total observed volatility (measured by squared returns) display evidence of considerable reactions to economic events and no apparent change in the trend.

The spillovers in volatility are subjected to further analysis to determine whether these changes are due to volatility shocks, or whether they are due to changes in the underlying latent volatility process. The spillover index for underlying latent volatility is found to be more stable and generally higher than the spillover index for total observed volatility. It is suggested that changes in the total observed volatility spillover index result from volatility shocks, whilst in certain instances country specific events (i.e. changes in the structure of an economy) may perpetuate changes in the trend of the underlying volatility spillover index.

Keywords: VAR, Spillover, Exchange Rates, Emerging Markets, South Africa, Stochastic Volatility

[1] In partial fulfillment for a Masters of Commerce in Economics, submitted on 10th February 2012 at the School of Economics, University of Cape Town, South Africa.

[2] Word count: 11 044

1. Introduction

In this paper we study the existence of spillover effects in advanced and emerging market currencies, with a particular focus on the South African rand. It has implications for policy makers and financial participants who are interested in understanding the dynamics and linkages of the foreign exchange (FX) market.

The construction of the initial spillover index follows the work of Diebold and Yilmaz (2009), who construct such an index for equity markets in different developed economies. They define a *spillover* as the share of the forecast error variance in one market that is caused by shocks to other markets. Such an index is calculated from the variance decompositions and impulse response functions that utilize a vector autoregressive (VAR) modelling framework.

When this method is applied to foreign exchange markets, we find similar results to those of Diebold and Yilmaz (2009), where the spillover in returns are relatively stable with a positive trend, and the spillover in total observed volatility is more erratic and reacts strongly to economic events. In the case of the South African rand, we find that the spillover index for returns rose from less than 20% in 2002 to over 60% in 2011.[3]

This methodological framework is then extended to distinguish between the effects of volatility shocks and changes to the underlying latent volatility process. To do so, we construct a stochastic volatility model that utilizes Bayesian inference and the particle filter of Liu and West (2001). We find that when we look for spillovers in the underlying latent volatility, South Africa was one of the few countries that experienced an increase in this underlying volatility spillover index. This could possibly be attributed to the continued opening up of the economy to foreign influence as well as recovering from a rand specific currency crisis in 2001.

The comparison of a spillover index for total observed volatility to a spillover index for underlying latent volatility is, to our knowledge, new to this field of research. As such, the paper contributes to the current body of spillover literature in two ways: first by applying the spillover index to the foreign exchange market including advanced and emerging market currencies, and secondly by providing new insights into the structure of volatility spillovers.

In what follows, section 2 contains a review of the current spillover literature. Section 3 contains a thorough overview of the methodology that is used to construct the spillover index and section 4 presents the results of this initial analysis. Section 5 contains an introduction to the framework for the stochastic volatility model and section 6 presents the results for the spillover index for underlying volatility. Lastly, we provide a brief summary of our findings and their implications.

[3] In a similar paper, Duncan and Kabundi (2011b) consider the construction of a spillover index for *different asset classes* (e.g. from bonds to equities and vice versa) in South Africa, using a different approach that utilized the generalized impulse response functions of Pesaran and Shin (1997).

2. Review of the foreign exchange and spillover literature

The importance of understanding movements in the FX markets can be motivated by several characteristics: firstly, exchange rates and their volatility is of huge importance to economic policy makers and financial participants. An unfavorable move in the exchange rate can quickly eradicate the entire profit made from a financial investment or trade of goods. In addition, exchange rate volatility, or uncertainty about future volatility, will influence the risk-return tradeoff in economic and investment activities and will have a direct impact on the price of derivative instruments that are based on foreign assets. Thus, the importance of volatility modeling and forecasting is particularly clear in the case of financial risk management as well as in "derivative pricing and hedging, market making, market timing, portfolio selection and many other financial activities" (Engle and Patton, 2001).

Exchange rates and exchange rate volatility also have an effect on economic and social factors. There is considerable evidence that exchange rate volatility has significant effects on productivity growth. For instance, Aghion et al. (2006) find that "higher levels of exchange rate volatility can stunt growth, especially in countries with thin capital markets and where financial shocks are the main source of macroeconomic volatility". This indicates a particular sensitivity to exchange rates in emerging market countries where the financial markets remain less developed and possibly also less liquid. In terms of less developed countries, Arize et al. (2000) find exchange rate volatility to negatively affect export demand both in the short run and long run for all thirteen less developed countries included in their study. However, economists seldom view low exchange rate volatility as a determinant of economic growth, but rather as a facilitating condition (Eichengreen, 2007).

The importance of understanding FX markets can also be attributed to its enormous size and high liquidity. A small shift in exchange rates may have huge economic consequences, particularly for small emerging markets. The size of the global FX market is rapidly growing from 2 trillion US dollars per day in 2004 to 4 trillion US dollars of *daily turnover* in 2010 (BIS, 2010). This is 78 times the total daily international trade in goods and services for 2010 (IMF, 2011). One explanation of this massive volume may be the so-called "hot potato trading" which refers to market makers (dealers) passing on unwanted positions in the FX market as part of their risk management (Lyons, 2001).

There is a growing literature on spillovers both across asset classes and across international borders. Engle et al. (1990) is often quoted as a pioneering paper in spillover research, in which they study spillovers in the US dollar / Japanese Yen exchange rate across world markets. They show that there is significant evidence of "meteor shower" type spillover effects, where volatility in the exchange rate during Tokyo trading hours spills over to volatility in the same exchange rate the following day in European and American trading hours (Engle et al., 1990).

King et al. (1994) point out that international equity markets go through periods of sustained comovements. "There are certain periods - the 1987 stock market crash is a conspicuous example - when markets seem to move in unison, and others when the correlation between them appears to be low" (King et al., 1994). It is found that this time-varying correlation of markets is hard to explain, or predict, with observable economic data. Only during short time periods can a small fraction of this correlation be explained by observed variables (King et al., 1994). However, despite periods of unison moves across markets, they do not find evidence of financial integration in their data set ranging from 1970 to 1988. Financial integration in this case was defined by the null hypothesis that the price of risk (risk premium) is identical across countries. This null hypothesis was rejected, an indication that the expected returns may be different across markets, given a level of volatility (risk).

A different approach was made by Forbes and Rigobon (2002) who argue that markets are highly *interdependent* in the sense that shocks to one market can have strong impacts on other markets. They point out examples such as the Asian Crisis in 1997 which clearly affected stock markets in America, Europe and Africa, as well as the 1994 drastic decline in Mexican markets which quickly spread to other Latin American stock markets. However, their findings indicate that the degree of comovements during times of crisis are the same as during times of stability (Forbes and Rigobon, 2002). This would be an argument against so-called *contagion*, narrowly defined as an increase in *comovement* (or *interdepence*) of asset prices during financial crises. This is distinct from an increase in *spillovers* during financial crises. A higher spillover index during financial crises would indicate increased cross-country linkages, but it says nothing explicitly about comovements. That is, linkages are increased as the variance of currencies is more likely to be explained by shocks to other currencies. This could be a symptom of increased comovements or interdependencies (contagion), but it does not prove contagion as defined above.

In a study of solely emerging Latin American equity markets, Edward and Susmel (2001) find the interdependency of volatility to be consistently strong, but they do not find evidence of contagion. That is, the interdependency of volatility does not increase during financial crises. In a subsequent paper, Forbes and Chinn (2003) study the explanatory power of direct trade linkages and financial linkages on the degree of comovement of equity and bond markets. On a sample from 1996 to 2000, their findings indicate that direct trade still had a greater impact on financial market interdependencies than financial linkages did (Forbes and Chinn, 2003). This may have changed in the years from 2000 to the global financial crisis of 2008. Cetorelli and Goldberg (2010) argue that increased globalization of US banks allowed domestic liquidity shocks in the USA to rapidly spread to capital markets across the globe: a strong indication that financial linkages may have been a more important channel of interdependency than trade linkages was.

Forbes and Rigobon (2002)'s case against contagion (defined as above) may potentially be weaker after the global financial crisis of 2008. Recent evidence shows that financial market interdepence across borders has increased after the global financial crisis (Eichengreen et al., 2009). Diebold and Yilmaz (2009) find the degree of spillovers in exchange rate *volatility* to be highly responsive to dramatic events such as the Asian Crisis and the global financial crisis, while they find spillovers in exchange rate *returns* to have been less responsive to financial / geopolitical events. Hence, there are no symptoms of contagion in return spillovers, but volatility spillovers do indicate potential symptoms of contagion. Duncan and Kabundi (2011a) also suggest that spillovers in volatility of world equity markets was particularly strong during the financial crisis. However, they find that the time between crises, particularly from 2001 to 2007, is characterized by "decoupling" of emerging market volatility from developed market volatility. That is, the spillovers in volatility from advanced equity markets to emerging equity markets is lower during periods of financial tranquility. One would suspect that foreign exchange markets (the focus of this study) will behave similarly to equity markets in these regards, especially since research has often found exchange rates to be a strong receiver of volatility from domestic equity markets (Bonga-Bonga and Hoveni, 2011; Duncan and Kabundi, 2011b; Diebold and Yilmaz, 2011) .

Despite the lack of evidence of contagion, the consensus and evidence is clearly in support of increased *spillover effects* during financial crises. The distinction between contagion and increased spillover effects during financial crises is quite blurry, and consequently most spillover studies have "sidestepped" (in the words of Diebold and Yilmaz (2011)) the contentious *contagion* debate. However, it was argued above that increased spillovers do indicate increased interdependencies, possibly a symptom of contagion.

Contagion being present or not, given the fundamental role of FX markets in financial risk management, one would expect a thorough understanding of these spillover effects to be of high importance during financial crises when (perceived) risk is the highest.[4] This brings us back to the objective of the paper. We first introduce the framework of the spillover index that is used in the initial investigation. The estimates are then presented accompanied by a brief analysis. We lastly introduce the stochastic volatility model and present the spillover index using an estimate of the latent volatility process.

3. Deriving the spillover index

The spillover index as suggested by Diebold and Yilmaz (2009) is simple to derive yet rigorous and replicable, which facilitates ease of understanding and improved transparency. It relies on a Cholesky decomposition which imposes several restrictions. In this paper the ordering is based on trade volume, which imposes the restriction that shocks to more traded currencies may have same-day effects on less traded currencies, but not the other way around. An alternative spillover index was derived in Diebold and Yilmaz (2011), based on the generalized impulse response functions (GIRF). Kim (2009) shows that the

[4]Of course the true risk was high also before the crisis, but at this time few may have been aware of the risk. This is naturally a huge flaw in risk management itself, as it only concerns the risk we are aware of, but not unexpected tail events. See Taleb (2010) for a thorough exposition of "the impact of the highly improbable".

GIRF from a shock to variable a is identical to the impulse response functions from an ordered VAR where variable a is ordered on top (Kim, 2009). Hence, the GIRF essentially allows shocks to all variables to have contemporaneous effects on all other variables (which implies certain inconsistencies as explained in (Kim, 2009)). This seems unnecessary in our case as we do not expect shocks to less traded emerging market currencies, such as the South African rand, to have contemporaneous effects on advanced country currencies such as the Euro. Therefore, we argue that the Cholesky decomposition is unproblematic and the best choice for our model of exchange rates. The GVAR was more appropriate in the case of Duncan and Kabundi (2011b) and Diebold and Yilmaz (2011) who studied spillovers across asset classes (in which it would be hard to justify a certain Cholesky ordering). Appendix D shows that the use of GVAR in this case may give a lower spillover index assuming that the coefficients in the model are positive.

The spillover index suggested by Diebold and Yilmaz (2009) is derived as follows. The primitive form of the VAR model is expressed as:

$$\mathbf{x_t} = \mathbf{\Pi} + \mathbf{\Gamma_0 x_t} + \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{\Gamma_i x_{t-i}} + \varepsilon_\mathbf{t} \tag{1}$$

where, $\mathbf{x_t}$ is a $(k \times 1)$ vector of currency variables, whereas each variable, x_t^i, is a time series with n observations of returns or volatility. The $(k \times k)$ $\mathbf{\Gamma_0}$ matrix assigns coefficients to the contemporaneous variables and must have a diagonal of zeros (as these are the coefficients on the contemporaneous dependent variables). The $(k \times k)$ $\mathbf{\Gamma_i}$ matrices assign coefficients to the ith lag, $\mathbf{x_{t-i}}$. Lastly, ε is a $(k \times 1)$ vector of errors that are assumed to be independently and normally distributed.

In the following we will for simplicity assume that the average returns are zero, and hence remove the constant from the model. The primitive model may now be rewritten in the standard form as:

$$\mathbf{x_t} = \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{B_0 \Gamma_i x_{t-i}} + \mathbf{B_0} \varepsilon_\mathbf{t}, \tag{2}$$

where

$$\mathbf{B_0} = [\mathbf{I} - \mathbf{\Gamma_0}]^{\mathbf{-1}} \tag{3}$$

With the contemporaneous variables subtracted from both sides of the equation, the model as expressed in equation (2) is identified and may be estimated by Ordinary Least Squares (OLS). Under the assumption of multivariate normal errors, this estimate coincides with the Maximum Likelihood estimator and is the efficient Generalized Least Squares estimator (Davidson and MacKinnon, 2009).

However, although the standard form of the model (2) is identified, the primitive model (1) remains underidentified. We therefore impose a Cholesky decomposition on the $\mathbf{B_0}$ matrix such that we can estimate the orthogonalized errors from (1).

Spillovers occur when shocks to one currency causes unexpected returns in another currency. By *unexpected* returns, we mean the forecast errors from modelling each currency as a moving average process. We assume that there are no spillovers in the *expected* returns, as expected returns are explained by the autoregressive nature (the momentum) of the time series. The forecast errors, on the other hand, must either be explained by exogenous shocks to the domestic currency or spillovers from other currencies. Therefore, the *variance* of the forecast error is also explained by shocks to the domestic currency or by spillovers from other currencies. Accordingly, the spillover index is defined as the *share* of the forecast error variance that is caused by shocks to other currencies. [5]

Hence, since we in this study are interested in the forecast errors, we may rewrite the model (2) as an infinite order Moving Average process, assuming that the currency returns and volatility are covariance stationary. (The $\mathbf{L^i}$ is a lag operator referring to the ith lag of the following variable):

$$(\mathbf{I} - \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{B_0 \Gamma_i L^i})\mathbf{x_t} = \mathbf{B_0}\varepsilon_{\mathbf{t}} \tag{4}$$

$$\mathbf{x_t} = [\mathbf{I} - \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{B_0 \Gamma_i L^i}]^{-\mathbf{1}} \mathbf{B_0}\varepsilon_{\mathbf{t}} \tag{5}$$

To simplify the notation, we define:

$$\mathbf{A(L)} \equiv [\mathbf{I} - \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{B_0 \Gamma_i L^i}]^{-\mathbf{1}} \mathbf{B_0}, \tag{6}$$

such that:

$$\mathbf{x_t} = \mathbf{A(L)}\varepsilon_{\mathbf{t}}, \tag{7}$$

and the forecast error ($\mathbf{e_{t+1,t}}$) from forecasting $\mathbf{x_{t+1}}$ at time t equals to:

$$\mathbf{e_{t+1,t}} = \mathbf{x_{t+1}} - \mathbf{E_t}(\mathbf{x_{t+1}}) = \mathbf{A(L)}\varepsilon_{\mathbf{t+1}}, \tag{8}$$

where $\varepsilon_{\mathbf{t}}$ is the orthogonalized errors with identity covariance matrix, $E(\varepsilon\varepsilon') = \mathbf{I_k}$. That is, $\varepsilon_{i,t}$ is a shock purely to variable i, but it may spill over to variable j according to the coefficients in the $\mathbf{A(L)}$ matrix.

[5] By using the variance, the sign of a shock and the sign of its effect on other currencies do not affect the spillover index.

Using an example of a two-variable, first order VAR model (say returns on the EUR/USD exchange rate and the ZAR/USD exchange rate) we have the forecast error ($\mathbf{e_{t+1,t}}$):

$$\begin{pmatrix} e_{1,t+1} \\ e_{2,t+1} \end{pmatrix} = \begin{pmatrix} a_{0,11} & a_{0,12} \\ a_{0,21} & a_{0,22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{pmatrix} \tag{9}$$

Now, thanks to the fact that $E(\varepsilon\varepsilon') = \mathbf{I_2}$, this gives a covariance matrix equal to:

$$E(\mathbf{e_{t+1}e'_{t+1}}) = \begin{pmatrix} a_{0,11} & a_{0,12} \\ a_{0,21} & a_{0,22} \end{pmatrix} \begin{pmatrix} a_{0,11} & a_{0,12} \\ a_{0,21} & a_{0,22} \end{pmatrix}' \tag{10}$$

$$= \begin{pmatrix} a_{0,11}^2 + a_{0,12}^2 & a_{0,11}a_{0,21} + a_{0,12}a_{0,22} \\ a_{0,11}a_{0,21} + b_{0,12}a_{0,22} & a_{0,21}^2 + a_{0,22}^2 \end{pmatrix} \tag{11}$$

Diebold and Yilmaz (2009) define *own variance shares* as the fraction of the forecast error variance from forecasting x_1 due to shocks to x_1 and the *cross variance shares* as forecast error variance in x_1 caused by shocks to the other variable, x_2. The cross variance is what the paper will refer to as the *spillover effect*. The *total spillover* equals the sum of spillover effects from the two variables on each other.

Hence:

$$\text{total spillover} = a_{0,21}^2 + a_{0,12}^2 \tag{12}$$

and the *spillover index* as suggested by Diebold and Yilmaz (2009) which reflects the share of the total forecast error variance (sum of forecast variance in all currencies at all forecast horizons) that is explained by total spillovers:

$$S = \frac{a_{0,21}^2 + a_{0,12}^2}{a_{0,11}^2 + a_{0,12}^2 + a_{0,21}^2 + a_{0,22}^2} \times 100 \tag{13}$$

$$S = \frac{a_{0,21}^2 + a_{0,12}^2}{trace(\mathbf{A_0A_0'})} \times 100 \tag{14}$$

This may be generalized to a multivariate case of several exchange rates and at an H-step ahead forecast horizons:

$$S = \frac{\sum_{h=0}^{H}\sum_{i=1}^{k}\sum_{j=1}^{k} a_{h,ij,i\neq j}^2}{\sum_{h=0}^{H} trace(\mathbf{A_hA_h'})} \times 100 \tag{15}$$

where $a_{h,ij}^2$ indicates the spillovers *from* currency j *to* currency i in the h'th step ahead forecast error.

In this paper we also estimate *individual* spillover indexes for each currency as well as a *regional* spillover index for groups of currencies to understand the direction of the spillovers.[6] In this case, the regional index is defined as the share of forecast error variance in region A that is explained by shocks to region B. This enables us to investigate the share of forecast error variance in emerging markets that is explained by shocks to advanced economies (and *vice versa*). This is achieved by ordering the currencies such that all advanced country currencies are ordered above the emerging market currencies.[7] This makes the notation (and programming) quite simple, as the first a currencies belong to region A (advanced economies), while the following $k-a$ currencies belong to region B (emerging markets). Hence, the index for region A equals:[8]

$$S_A = \frac{\sum_{h=1}^{H}\sum_{i=1}^{a}\sum_{j=a+1}^{k} a_{h,ij}^2}{\sum_{h=0}^{H}\sum_{i=1}^{a}\sum_{j=1}^{k} a_{h,ij}^2} \times 100 \tag{16}$$

And for region B:

$$S_B = \frac{\sum_{h=0}^{H}\sum_{i=a+1}^{k}\sum_{j=1}^{a} a_{h,ij}^2}{\sum_{h=0}^{H}\sum_{i=a+1}^{k}\sum_{j=1}^{k} a_{h,ij}^2} \times 100 \tag{17}$$

And similarly for an H-step ahead forecast with k variables we calculate the *individual* currency spillover index for currency i as:

$$S_i = \left[1 - \frac{\sum_{h=0}^{H} a_{h,ii}^2}{\sum_{h=0}^{H}\sum_{j=0}^{k} a_{h,ij}^2}\right] \times 100 \tag{18}$$

It should be clear from (18) that the *individual* spillover index reflects the share of the variance of the forecast errors on currency i that is explained by shocks to all other currencies in the model. While the *regional* spillover index reflects the share of forecast error variance in currencies included in region B that is explained by shocks to currencies in region A.

[6] Unfortunately Diebold and Yilmaz (2009) did not include such an analysis in their paper.

[7] A lower Cholesky decomposition is imposed, such that shocks to the “top” variable, advanced economies, may have contemporaneous effects on “lower” variables, emerging markets. While shocks to “lower” variables cannot have contemporaneous effects on the “above” variables.

[8] The forecast horizon (h) in $S_A$begins at 1 as the contemporaneous shocks are restricted to only happen in the direction from region A to region B.

Part I
Initial Investigation

4. Data and estimation

The following section will present the data and the estimated spillover index.[9] For transparency we go through each of the steps in the estimation procedure before we discuss the results.

4.1. Exchange rate data

The model includes three groupings of currencies, chosen according to their trade volume and relevance. It is desired for the study that the respective exchange rate regimes are floating, preferably independently, but managed floats are also of interest. In the case of managed floats, or periods of reserve bank intervention, we would expect this to reduce the spillover index for the respective currency as this clearly would be a country specific innovation that cannot be explained by spillovers from other currencies. We classify the currency regimes according to the IMF's *Annual Report on Exchange Arrangements and Exchange Restrictions* (IMF, 2009).

The US dollar was involved in 85% of all FX transactions in 2010, down from its peak of 90% in 2001, while the Euro was involved in 39% of all transactions in 2010, up from 23.5% in 2001. In declining order of volume, the most traded currencies after the US dollar and the Euro are the Japanese yen, pound sterling, Swiss franc and the Australian dollar. Of what may be considered emerging market currencies, the most traded are the Hong Kong dollar, the Korean Won, Singapore dollar, Mexican peso, Indian rupee, Russian rouble, Chinese remnibi, Turkish lira, the South African rand and the Brazilian real (BIS, 2010). The paper does not provide an outright definition of *emerging markets*, but we rather choose five countries included in the widely quoted MSCI Emerging Markets index (MSCI, 2011).[10]

All data utilized in this research was gathered from Thompson Reuters Datastream through the WM/Reuters channel. These exchange rates are quoted at or around 16:00 in London. "This time reflects the middle of the 'global day' and the time of highest liquidity in the foreign exchange market" (Reuters, 2011). It is important for the study that all exchange rates are quoted at the same time, as we wish to avoid the "meteor shower" effects that were demonstrated by Engle et al. (1990).[11] We use six advanced country currencies, four

[9]The spillover index is estimated in *Mathworks Matlab R2011a* with code written by the authors of the paper.

[10]Appendix B includes individual spillover indexes for an additional 15 emerging market currencies.

[11]Meteor showers refer to volatility spillovers across time zones, e.g. volatility during trading hours in Tokyo may spill over to volatility the following day during trading hours in London or New York (Engle et al., 1990).

Table 1: Exchange Rate Regimes - Selected currencies ordered according to trade volume

Code	Currency	1998	2001	2004	2007	2010
USD	US Dollar	IF	IF	IF	IF	IF
EUR	Euro	IF	IF	IF	IF	IF
JPY	Japanese Yen	IF	IF	IF	IF	IF
GBP	British Pound Sterling	IF	IF	IF	IF	IF
AUD	Australian Dollar	IF	IF	IF	IF	IF
CHF	Swiss Franc	IF	IF	IF	IF	IF
CAD	Canadian Dollar	IF	IF	IF	IF	IF
KRW	Korean Won	IF	IF	IF	IF	IF
MXN	Mexican Pesos	IF	IF	IF	IF	IF
INR	Indian Rupee	IF	MF	MF	MF	MF
ZAR	South African Rand	IF	IF	IF	IF	IF
BRL	Brazilian Real	MF	IF	IF	IF	IF
NGN	Nigerian Naira	MF	MF	MF	MF	MF
EGP	Egyptian Pound	MF	P	MF	MF	MF
KNS	Kenyan Shilling	MF	MF	MF	MF	MF

IF: Independent Float, MF: Managed Float, P: Peg or Currency Board
Source: *IMF Annual Report on Exchange Arrangements and Exchange Restrictions, 1998-2010,* and *Bank of International Settlements Triennial Central Bank Survey 2010.*

emerging market currencies (including South Africa) and five African currencies (also including South Africa); all with the US dollar as base currency. The advanced country currencies are chosen according to trade volume as reported by Bank of International Settlements in their *Triennial Central Bank Survey* (2010). We then select four of the most traded emerging market and African currencies according to trade volume, relevance and exchange rate regime. The sample thus consists of daily exchange rate data on 14 currencies from 15th November 1997 to 15th November 2011. The selected currencies are listed in table 4.1 and are ordered by trade volume (the same order as in our Cholesky decomposition).

Given the Cholesky ordering based on trade volume, we have implicitly imposed the assumption that shocks to more traded currencies may have contemporaneous (same day) effects on less traded currencies, but shocks to less traded currencies cannot have contemporaneous effects on more traded currencies. However, shocks to all currencies may affect all other currencies at a lag. For reference, Appendix A includes plots of the *overall* index for a variety of different orderings. As the reader will see, the plotted indexes are remarkably similar for each ordering of the variables, perhaps with the exception of *order 2* which is the reverse order of what was used in the main estimations. [12]

The exchange rates are converted into daily continuously compounded returns by using the first difference of the natural logarithm of the exchange rates. To estimate the volatility spillovers we first need an estimate of volatility in each of the currencies. Diebold and Yilmaz (2009) used a volatility estimate for equities based on the difference between open-close prices and bid-ask spreads, while Diebold and Yilmaz (2011) use the difference between daily high and low prices. We do not have access to this type of data and follow Duncan and Kabundi (2011b) who use squared returns as a measure of total observed volatility. [13] Although this is a common proxy, it has been suggested that it is a poor estimate of realized volatility as it is "plagued by large idiosyncratic errors" (Andersen et al., 2006). That is, it captures and amplifies all day-to-day noise in the exchange rates and will consequently have a very high *noise-to-signal* ratio (Andersen et al., 2006). In a subsequent analysis we therefore investigate the spillovers in volatility by estimating a separate spillover index using an estimated stochastic volatility model for the *underlying* latent volatility process. The advantage of estimating the spillover index for both volatility proxies is that we are able to distinguish between effects of volatility shocks and changes in the underlying volatility process.

4.2. Lag length

Given the fact that the spillover index derived above is based on the residuals from a VAR(p) model, we must first ensure that the VAR model is appropriately specified. We have already justified the use of variables and their ordering, but we must still determine the number of lags (p) for each variable. Given that the model has a large multivariate structure, the addition of lags will quickly erode the degrees of freedom. However, too few lags result in a misspecified model. To find the optimal lag structure, we estimate the model for the highest realistic order and reduce the order incrementally. For each estimation we store the vector of residuals and calculate the Maximum Likelihood (ML) estimate of the residual covariance matrix.

The ML estimator of the covariance matrix is calculated according to equation (19) where $\hat{\mathbf{U}}_{\mathbf{t}}$ refers to the row vector of residuals from each equation at observation, t, and T is the last observation (Davidson and MacKinnon, 2009) :

$$\hat{\mathbf{\Sigma}} = \frac{1}{T}\sum_{t=1}^{T}\hat{\mathbf{U}}_{\mathbf{t}}'\hat{\mathbf{U}}_{\mathbf{t}} \tag{19}$$

[12]This reverse order is completely unrealistic since the Kenyan shilling is allowed to have contemporaneous effects on all currencies while the Euro can have no contemporaneous effects on any currencies.

[13]Note that since the variables are expressed as returns, they have in effect been standardized (they have no units of measurement). Hence, there is no need to convert the squared returns into standard deviations.

We then use the determinant of this estimated covariance matrix ($|\hat{\mathbf{\Sigma}}|$) to calculate the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC) for multivariate models:

$$AIC = T * ln|\hat{\mathbf{\Sigma}}| + 2 * N \tag{20}$$

$$SBC = T * ln|\hat{\mathbf{\Sigma}}| + N * ln(T) \tag{21}$$

where N is the total number of parameters in the model, including intercepts and all lags. Both the SBC and the AIC were found to have the largest negative value for the *second order* VAR model for exchange rate *returns* (that is, two lags of each variables are indicated to be optimal). In the VAR model for the *volatility* of exchange rates we find the AIC to suggest a lag order greater than 10, while the SBC suggests volatility to also be best modeled by a *second order* VAR. Hence for simplicity, and to save degrees of freedom, we will in the remainder of the paper estimate all return and volatility VAR models as VAR(2).

4.3. Static estimates and rolling windows

As a very fist step to initiate the study we estimate a static spillover index that is assumed to be constant over the entire sample. At this point we introduce four indexes that we will use throughout the remainder of the paper: (1) the overall spillover index, (2) the Africa spillover index (excluding South Africa), (3) the emerging markets Spillover index (including South Africa) and (4) the individual currency spillover indexes. The meaning of these four measures must be clear before we continue: The overall index reflects the sum of spillovers into all currencies as a share of total forecast error variance. The emerging markets index reflect the percentage share of forecast error variance in the emerging market currencies that are caused by shocks to advanced economies. The Africa spillover index reflects the similar spillovers from advanced economies *and* emerging markets into African currencies. Lastly, the individual indexes measure the percentage share of the forecast error variance in the respective currency that is explained by shocks to all other currencies.

We first estimate a static spillover index over the entire sample. We estimate the *overall spillover index* to be 23.25% for returns and 7.82% for volatility (squared returns) at a twenty-day forecast horizon. That is, 23.25% of all variance in the forecast error of the average currency is explained by spillovers from shocks to other currencies. 7.82% of variance in the forecast error of volatility in the average currency is explained by shocks to other currencies. Enders (2010) suggest that in several empirical studies it is found that the forecast error variance is mostly explained by its *own* shocks at short forecast horizons, while at longer forecast horizons the variable's own shocks explain a smaller proportion of its error variance. This corresponds to the findings of Diebold and Yilmaz (2009) as well as our own findings reported in figure 1 which illustrates that the estimated static spillover index indeed is a nondecreasing function of the forecast horizon. As Enders (2010) notes, "we would expect this pattern if shocks

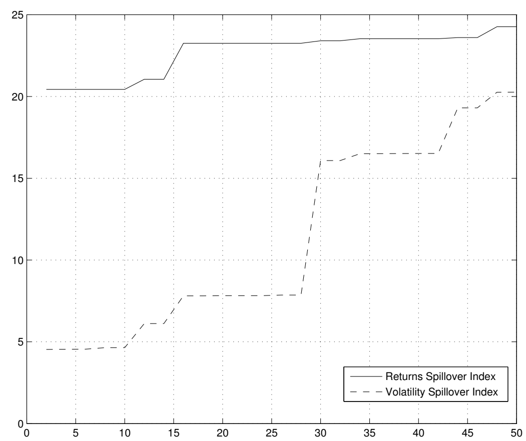

Figure 1: Static overall spillover index for returns and squared returns (volatility) calculated with forecast errors from 1 to 50 steps-ahead forecast horizons

[to currency z] had little contemporaneous effect on [currency y] but acted to affect the y_t sequence with a lag" (Enders, 2010). The fact that longer forecast horizons indicate a greater spillover index means that the spillovers from one currency to another do not happen immediately, but gradually over the course of several days. Figure 1 indicates that this is certainly the case for volatility spillovers.

Since it is unlikely that the magnitude of spillover effects has been constant over the entire sample period of 14 years, we need to estimate a time-varying spillover index by using a shorter sample window. We then roll this sample window forward, whilst holding the sample size constant, from the beginning to the end of the sample. We have used two different sample sizes for the rolling window: (1) 260 days (1 year) and (2) 780 days (3 years).[14] For reference, Diebold and Yilmaz (2009) used a rolling 200-week (approximately 4 years) window for their estimations on weekly returns. The implications of the different sampling windows are that a large window will give a smoother dynamic index series and more accurate estimates (as there are more observations / degrees of freedom in each estimate), but it does imply an assumption that the true index did not change during the window period. A smaller sample may give a less accurate estimate, but they provide an indication of how the index fluctuates in the shorter term.

[14]The time-varying spillover index was also estimated on a 160 day rolling window, but due to the short window / small sample, the resulting index contained too much noise to provide any useful information and was therefore omitted from the analysis.

5. The spillover index for returns and total observed volatility (squared returns)

Figures 2 - 5 display the three regional indexes introduced above as well as the South African spillover index. Each index is plotted with two different forecast-horizons, ten days (H=10) and two days (H=2). The graphs report the spillover index at the end of the sampling window so that the spillover index reported for, say, January 2008 reflect a static index estimated over the sample from January 2007 - January 2008 with the *one year* window and a sample from January 2005 - January 2008 with the *three year* window. As expected the larger sample windows give a smoother estimated dynamic spillover index and a better picture of the longer term development of the index. Therefore, we choose to focus the latter part of the paper on the *three year* sample windows, and the remaining *individual* currency indexes are only plotted with this window size and are estimated with a ten day forecast horizon (H=10). It should be noted that all figures displaying a *volatility* index are for now estimated on a squared returns volatility proxy (the corresponding indexes estimated from a stochastic volatility model will be reported in section 6).

Inspection of figures 2 - 5 reveals a tendency of volatility to display more extreme changes in spillovers than returns do. This corresponds nicely to Diebold and Yilmaz (2009) who found the same interesting pattern. Return spillovers have increased gradually over the entire sample period, at the highest growth rate prior to the global financial crisis and now seem to have stabilized at a relatively high level of 45% overall, and as much as 60% in South Africa. Volatility spillovers display no apparent trend, but responded to the financial crisis by jumping up by 30 percentage points before it stabilized at approximately 10 percentage points above the pre-crisis level. The fact that volatility spillovers increase sharply (and become more volatile themselves) during the financial crisis is no great surprise and similar results have been reported frequently in past research as noted by Duncan and Kabundi (2011b) and the references therein.

Another striking characteristic of our results is that the *overall spillover index*, the *emerging market spillover index* and the *South Africa spillover index* all behave fairly similarly. Hence, when there is a change in the magnitude of spillovers in general, this will most likely include a similar change in the spillover to emerging market currencies and to the South African rand. The African spillover index, on the other hand, appears to move quite independently of the others. Furthermore, we note the strong and steady increase in the spillover index for the South African rand returns and volatility from less than 20% ten years ago to 60% the past three years. In other words, 60% of the forecast error variance is now due to shocks to other currencies than the rand.

5.0.1. Return spillovers

Similarly to our study, Diebold and Yilmaz (2009) find a steady increase in the return spillover index in their sample from 1995 to 2007. They particularly emphasize the point that the return spillovers never declined to the low levels

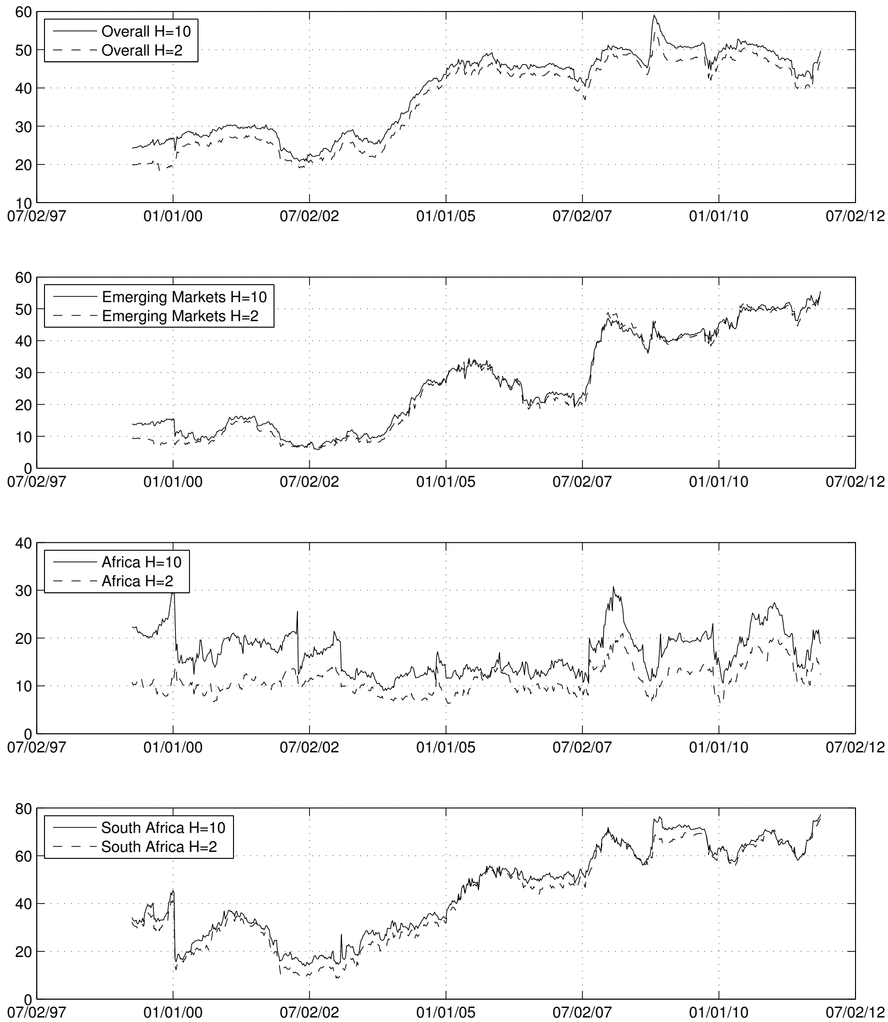

Figure 2: Spillover index for returns estimated with 1 year (260 day) rolling window

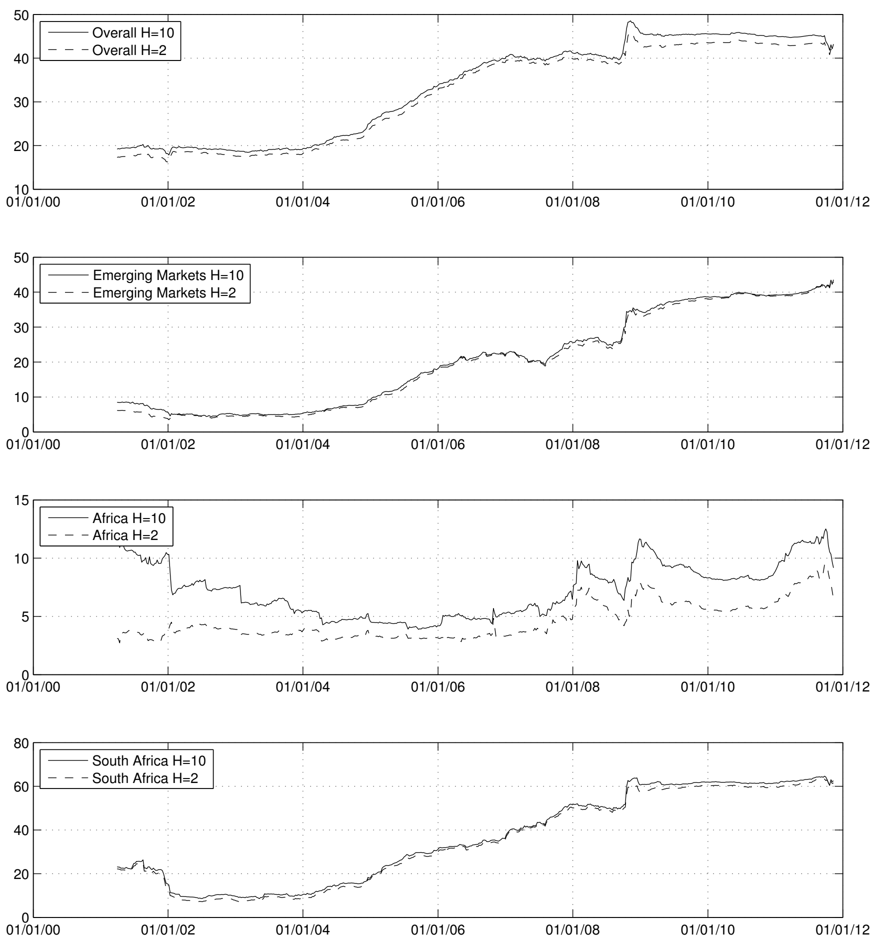

Figure 3: Spillover index for returns estimated with 3 year (720 day) rolling window

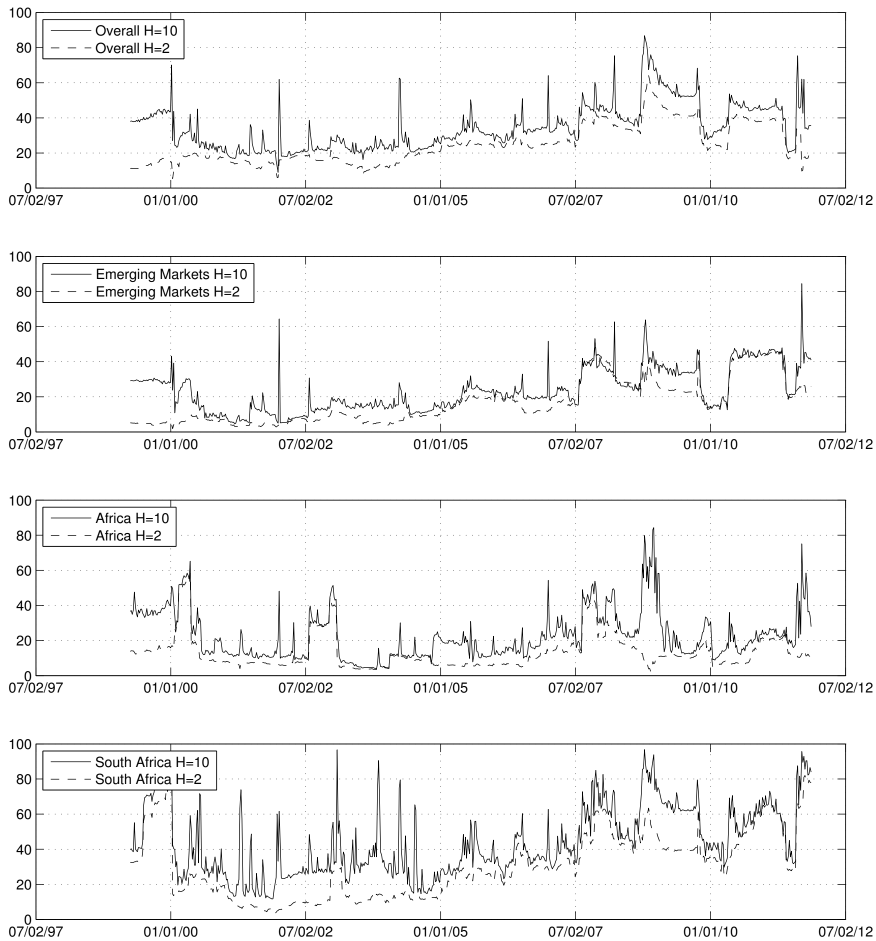

Figure 4: Spillover index for volatility estimated with 1 year (260 day) rolling window

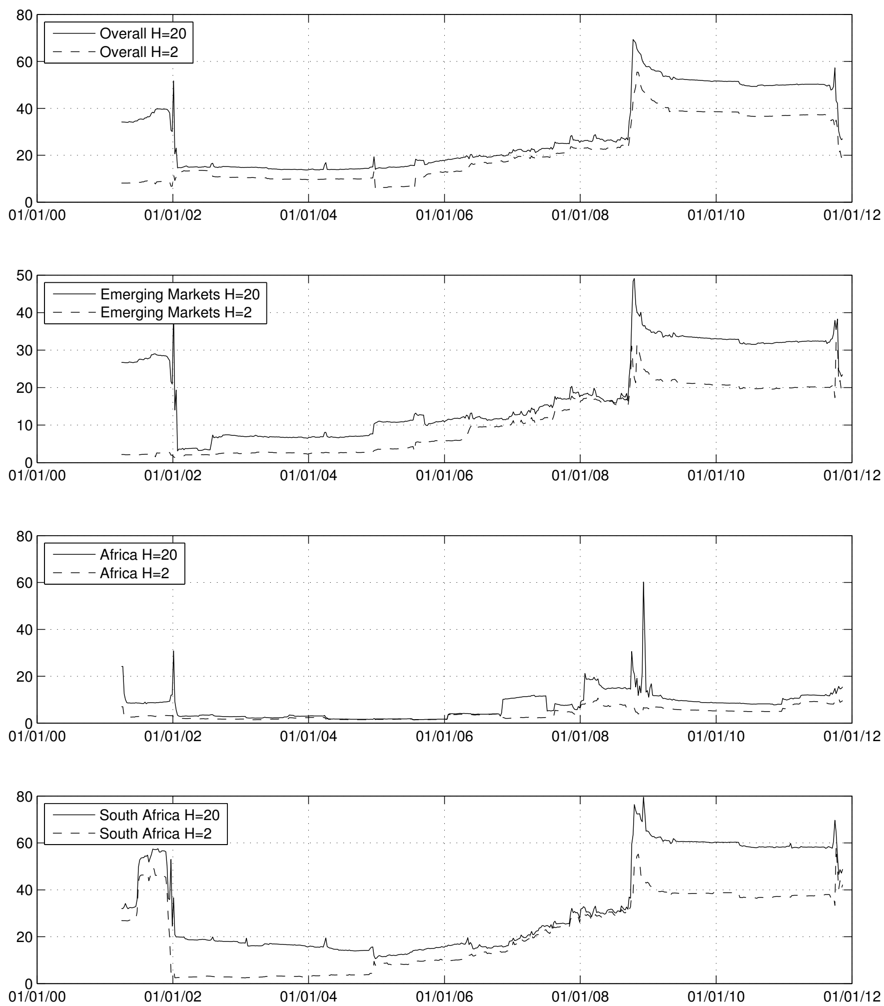

Figure 5: Spillover index for volatility estimated with 3 year (720 day) rolling window

seen early in their sample. This, according to their argument, is "consistent with a maintained increase in financial market integration" (Diebold and Yilmaz, 2009). They then note that the increasing trend in the return spillovers have become steeper in recent years (prior to the financial crisis). As may be seen from the figures (2) and (3), more or less the same is found from our estimates on return spillovers in exchange rates. We find that the return spillover index is consistently higher in the second half of our sample compared to the first half, possibly due to the "maintained increase" in financial integration (Diebold and Yilmaz, 2009) . And according to our 3-year rolling sample, which corresponds the closest to the 200-week rolling window in Diebold and Yilmaz (2009), the growth rate in return spillovers is indeed the fastest prior to the crisis, from 2004 to 2007, again similar to the findings of Diebold and Yilmaz (2009).

We find the South African spillover index to closely resemble the overall spillover index up until this point in 2007 when they both are just below 40%. However, a crucial development occurs after 2007, where the overall spillover index appears to stabilize at this level around 45%; the same may be said for the emerging markets spillover index; while the South African spillover index continues to grow at a slightly decreasing rate all the way to the end of our sample where it appears to settle around 60%. Spillovers in the South African rand have gone from being below the average of our sample to 15 percentage points above the average. It was found by Duncan and Kabundi (2011b) that spillovers between different South African asset classes are much greater than what Diebold and Yilmaz (2011) found between different asset classes in the USA. Duncan and Kabundi (2011b) attribute this to South Africa's status as an emerging market and as such being less developed than the USA.

An inspection of the figures in Appendix B reveals that the movement of the South African spillover index is indeed similar to other emerging market currencies, such as the Brazilian, Mexican, Korean, Hungarian, Russian, Polish and Turkish indexes (figures B.20 - B.21). There is no obvious common denominator between these economies (such as size, openness or policy) other than the fact that they all fall in the category of emerging markets. The fact that the South African spillover index is greater than the *emerging markets spillover index* in figure 3 may be partly attributed to the fact that the South Africa index includes shocks from other emerging markets whereas the emerging markets index does not (it only includes spillovers from advanced economies into emerging markets).

The typical explanation of increasing spillovers has been increased financial integration (Duncan and Kabundi, 2011b; Diebold and Yilmaz, 2009). This argument is supported by the fact that the period where the spillover index has displayed the fastest growth rate was characterized by increasing globalization of US banks (Cetorelli and Goldberg, 2010), increased synchronization of business cycles (Kose et al., 2008) and increased financial global interdependencies (Eichengreen et al., 2009): all consistent with increased spillover effects. This may also explain the consistently high growth rate of the South African Spillover index for returns. Bonga-Bonga (2009) uses a covered interest rate parity based model of financial integration to estimate the time varying integration of South Africa. He finds evidence that during the time period from 1993 to 2008 South Africa has become "progressively more integrated into the world financial market, despite short term deviation" (Bonga-Bonga, 2009).

One may expect that increased integration of South Africa in the world's financial markets would cause an increased trade volume of the rand. But growth in trades involving the South African rand has not increased faster than the overall FX market, with the rand accounting for 0.7% of all FX turnover in 2010, equal to the ratio in 2004 (and lower than in 2001 and 2007 when for both years it accounted for 0.9% of all FX turnover) (BIS, 2010). This must mean that the same trade volume has changed its characteristics: fifteen years ago it was largely influenced by South African specific factors, according to our estimates, while today the same trade volume is largely influenced by global factors.

In terms of spillovers from other *asset classes*, Duncan and Kabundi (2011b) find both bond markets and equity markets to have periods of high spillover effects on the South African rand. For the United States, Diebold and Yilmaz (2011) find exchange rates to be the biggest receiver of spillover effects from other asset classes with up to 10% of variance in exchange rate volatility being due to shocks to other asset classes. The implication of this is that shocks to equity and bond markets may spill over to the currency of the respective country, but also to other international asset markets given the high presence of international equity market spillovers (approx. 60% as shown in Diebold and Yilmaz (2009)). This may of course again spillover from these international bond and equity markets to their respective currencies. Given that other asset markets are not included in our model, these effects will take the form of forecast errors and hence indicate spillover effects between currencies. That is, the initial shock on, say, EU equity markets may first spill over to the Euro and later have a lagged effect on the rand through spillovers from EU equity markets to South African equity markets to the rand. This would imply that increased correlation between South African equity markets and global equity markets should increase the spillover index for the rand, *ceteris paribus.*

Duncan and Kabundi (2011b) do not include commodities as an asset class in their model. Frankel (2007) finds that an index of prices of South African mineral exports (mostly gold and coal) are highly significant explanatory variables of the rand. Hence, our ex ante expectations would be for the rand to be highly influenced by shocks to, say, the Australian dollar as they both may be described as commodity currencies (Frankel, 2007). For curiosity, we note that a simple OLS regression over the entire sample period of rand *returns* on Australian dollar returns yields an R-squared of .2615 while an OLS regression of the rand on the entire sample of exchange rate returns yields an R-squared of .294. In other words, changes in the Australian dollar alone explain almost as much as the entire sample of exchange rates (including the Australian dollar) does combined. We do not argue that changes in the Australian dollar cause changes in the rand, but rather that they both depend on a common factor, such as the price of their mineral exports, as shown by Frankel (2007). This could potentially mean that the continued increase in the rand spillover index after 2007 may have been due to high dependency on commodity prices, and that shocks to these prices are captured by the Aussie dollar. This would not be surprising given the extreme growth in gold prices between 2007 and 2011, displayed in figure (6).

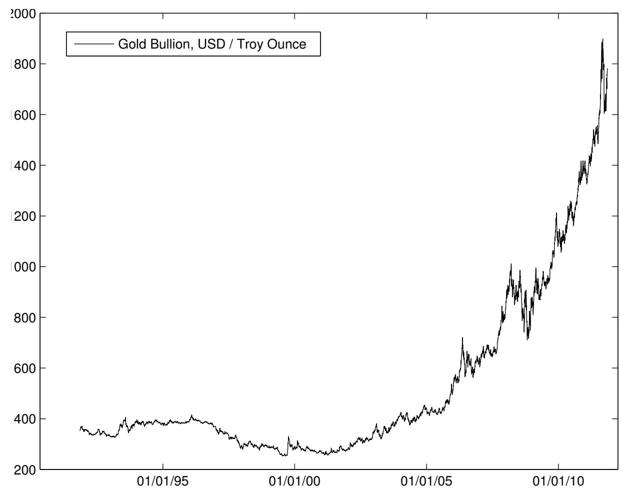

Figure 6: Price of gold bullion, US Dollars per Troy Ounce

5.0.2. Volatility spillovers

In terms of volatility spillovers (still measured by squared returns), there are much more short term abrupt movements across all indexes. As with returns we find very similar patterns for all volatility indexes, but a higher degree of spillovers in the South African rand after the global financial crisis. We suspect that the high levels of all spillover indexes at the beginning of the sample (remember, the index is reported at the end of the sample window) is caused by the Asian crisis and the dot-com bubble in the late 1990's and early 2000's. Furthermore, the large spike in spillovers in 2008 is caused by the global financial crisis. These responses to financial crises are particularly evident with the 720-day window as displayed in figure (5). For all currencies, except for the Africa index, volatility spillovers have been higher after the crisis than before, with a gradual increase in spillovers leading up to the financial crisis and then a sudden jump in 2008 to a new and higher level that has remained persistent since then. This is again precisely as one would expect based on the existing literature on financial integration and the financial crisis. [15] This also concurs with

[15] See Eichengreen et al. (2009).

the notion that volatility correlations increase during times of financial stress. By estimating the correlation coefficient between each currency at a rolling one year window we can calculate and plot the average correlation coefficient as was done in King et al. (1994). We see from figure 7 that the financial crisis of 2008 greatly increased the average correlations in currency volatility. Our evidence from the spillover index would suggest that a share of these correlations are explained by shocks to one currency spilling over to other currencies.

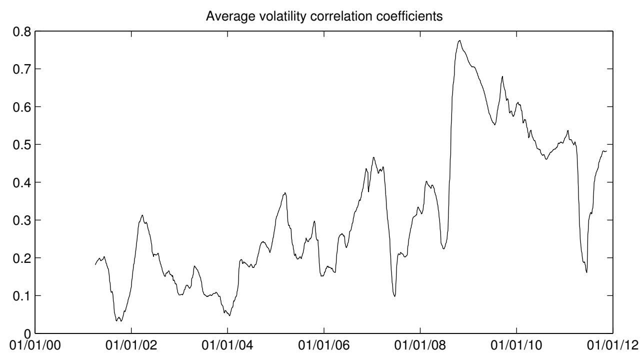

Figure 7: Rolling equally weighted average of correlation coefficients in currency volatility

In contrast to the emerging market spillover index and overall spillover index, the Africa index is consistently extremely low (less than 20%) and only displays a very short term reaction to the financial crisis. This again is most likely a reflection of limited integration of these African economies in the world's financial markets (remember that South Africa is not part of the Africa index).

When we compare the volatility indexes to the return indexes, we notice that not only do they react more abruptly to financial events, but they also display a wider gap between indexes estimated at different forecast horizons (H=2 and H=10). This indicates that whatever causes a change in volatility in one currency affects volatility in the other currency up to 10 days later. Evans and Lyons (2008), both renowned researchers in the financial microstructure field, find that approximately 30% of volatility in exchange rates can be attributed to the arrival of macroeconomic news. A logical extension, maybe attributed to that finding, would be that increased volatility during the financial crisis is partly due to an increased rate of macroeconomic news arrivals.

Evans and Lyons (2008) also find that the peak in return variance caused by the arrival of news is reached after 60-90 minutes. It is thus hard to explain why spillovers may have lags of multiple days, but one reason could be the indirect channel through which the news travel. By using the same example as earlier, a shock to EU equity markets may affect volatility of the Euro. This shock, may not have a direct impact on the rand, but it may very well affect volatility in South African equity markets through equity market spillovers. And this volatility in equity markets may very well spill over to the rand, as showed by Duncan and Kabundi (2011b). Hence, there may be a lag due to the indirect channel of the news flow, where at every stage traders must adapt their expectations to the arrival of unexpected news and a market consensus must be reached before prices stabilize.

In summary, the estimated returns and squared returns spillover indexes were found to behave in a strikingly similar manner to those estimated by Diebold and Yilmaz (2009) for world equity markets. In particular we found the same characteristic of returns to increase gradually over the sample with no abrupt movements, while volatility spillovers displays no clear trend but strong reactions to financial events. The following sections in Part II will proceed with a more thorough investigation into the structure of these volatility spillovers.

Part II

Investigating Volatility Spillovers

As most asset types, floating exchange rates are characterized by strong dependency in the price variance, such that one large movement is often followed by another large movement. Evidence of such characteristics in asset prices has been reported in papers as early as Mandelbrot (1963). Since then, volatility modeling has become an enormous area of research in financial economics and this has resulted in several stylized facts (Engle and Patton, 2001). These stylized facts include the observed *volatility clustering*, but also suggest that volatility is *mean reverting*, that price shocks (innovations) may have asymmetric effects on volatility and that volatility often may be explained by other *exogenous* variables (Engle and Patton, 2001). Volatility clustering has one very important implication: it means that the *expected* (near future) variance in asset prices *conditional* on past observed variance is different from the *unconditional expectations*. The discovery of these stylized facts of volatility processes have enabled researchers to estimate accurate volatility forecast on an interdaily basis (Andersen and Bollerslev, 1997; Engle and Patton, 2001).

According to Sarno and Taylor (2002), modeling of volatility in foreign exchange markets have in the past been dominated by autoregressive conditional heteroskedasticity (ARCH), first proposed by Engle (1982), and generalized ARCH (GARCH) models, first proposed by Bollerslev (1986). However, volatility is not directly observed and this may be acknowledged in a state-space model where volatility is included as an unobserved state variable including its own stochastic element (Andersen et al., 2006). This, more computationally costly, but also more empirically realistic approach, has been found to provide in-sample fit of the same quality as more heavily parametrized GARCH models (Kim et al., 1998). Stochastic volatility models have long been used in option pricing (dating back to the mid 1980s) and has since taken "center stage in econometric analysis of volatility forecasting" (Shephard and Andersen, 2008). In this part of the paper we will utilize a stochastic volatility model to distinguish between volatility shocks (measured by squared returns) and changes in the underlying latent volatility process.

Stochastic volatility models adopt a state-space framework where the state of the latent volatility may be estimated with the use of a Kalman filter. However, where the latent process exhibits nonlinear or non-Gaussian properties, the application of the Kalman filter is not suitable (in a model with unknown parameters) (Petris et al., 2009). Whilst Markov Chain Monte Carlo (MCMC) algorithms can to some extent deal with unknown parameters in nonlinear and non-Gaussian models, they are not suited for sequential estimations as the entire Markov Chain must be reestimated after the arrival of new data (Petris et al., 2009).[16] Hence, when using these methods one would either need to approximate the latent process in a linear Gaussian setting, using the Kalman filter, or one would need to employ an 'off-line' non-sequential MCMC filter to estimate the nonlinear, non-Gaussian model for the entire fixed sample.

It has been suggested that more recent developments that use Sequential Monte Carlo (SMC) algorithms are better suited to models that encounter unknown parameters, nonlinearities, non-Gaussian data and online inference (Petris et al., 2009). Early work by Pitt and Shephard (1999) on the auxiliary particle filter circumvents the problem of nonlinearities and non-Gaussian state variables by using importance sampling to assign weights to each randomly generated particle, where an auxiliary variable is sampled for each of these particles. This algorithm can then choose the particles that give higher likelihood values for observing the actual data conditional on the state variables (Prado and West, 2010; Petris et al., 2009). However, the auxiliary particle filter is not suited to models that require parameter learning, that is a concern in many applied research problems. Liu and West (2001) present a solution to this problem by including a vector of unknown parameters in the target distribution, where one draws a set of random particles at each data point from the prior distribution for both the state variable and the unknown parameters (in which one allows an artificial evolution of the static unknown parameters).[17] This implies that the Liu and West (2001) algorithm, which is regarded as a popular SMC method, would allow us to sequentially estimate the joint probability distribution of the state of the *latent volatility* and the unknown parameters, all conditional on the observed variable, which in this case is derived from the *returns* on exchange rates.

This methodology is of course fundamentally different from the methodology used to estimate the spillover index in the sense that it takes a Bayesian approach rather than the frequentist approach of Ordinary Least Squares and Maximum Likelihood. In the following section we provide a brief introduction to Bayesian inference, the Liu and West (2001) algorithm as well as defining our stochastic

[16] The Kalman filter effectively bases estimates of the state of the latent process at time t on the posterior distribution at time $t-1$ in a relatively straightforward manner.

[17] See, Prado and West (2010) and Petris et al. (2009) for a detailed textbook description on the application of the Liu and West (2001) filter.

volatility model. We then proceed by presenting the estimated underlying latent volatility and the resulting spillover index for underlying volatility.

5.1. Particle filters in estimation of stochastic volatility

The Bayesian approach is different from the frequentist's in that it incorporates the researchers uncertainty about the parameter of interest by assigning it a probability distribution. One then estimate the probability distribution conditional on the information in the data set (Petris et al., 2009). [18] Therefore, assume we have data on a vector of variables, y, and we wish to estimate the vector of parameters, θ, that explains the true relationship between the variables in y. In other words, we wish to find the probability distribution of θ *given* the observed data series, y: $\pi(\theta|y)$.

According to the familiar Bayes' formula, we have:

$$\pi(\theta|y) = \frac{\pi(y|\theta)\pi(\theta)}{\pi(y)} \tag{22}$$

Hence, what we need in order to find the distribution of interest is the marginal probability distribution of θ and y as well as the conditional distribution of y given θ. It should be noted that for sufficiently large sample sizes and with normally distributed data, the posterior distribution of θ asymptotically approximates a normal distribution around the Maximum Likelihood estimate. But despite its theoretical simplicity, the distributions can in practice be extremely difficult to calculate analytically. However, by formulating the stochastic volatility model in a state-space framework, one may use MCMC or SMC to approximate the state-space models. Both MCMC and SMC can handle nonlinearities (that naturally arise in a volatility model), but the SMC is superior for online sampling where the model must be re-estimated continuously as new data arrives (Lopes and Tsay, 2011). This would for example be of high importance in the financial industry where a model is desired to update at every discrete datapoint that is observed throughout the course of the day.

Now, in order to estimate the stochastic volatility of currency returns we define a simple state-space model and estimate the state equation by the Liu and West (2001) filter. We define the return series as y_t to be a function of the stochastic volatility series, h_t, and independently distributed innovations, e_t. The stochastic volatility model that we have used contains a constant and is assumed to be autoregressive of order one, where the volatility equation contains its own stochastic innovations, v_t (which distinguishes this stochastic model from an ordinary ARCH model). The innovations to returns, e_t, are assumed to have a students-t distribution (due to periods of high return variance, see figure 8 and C.23) and the innovations to volatility, v_t, are assumed to be independently

[18] The frequentist approach tries to estimate the objective and true value (which of course has probability of one and zero variance) and one always makes a prior assumption about the true data generating process.

normally distributed. This framework is based on Jaquier et al. (1994), Kim et al. (1998) and Lopes and Tsay (2011).

$$y_t = \sqrt{h_t} e_t \tag{23}$$
$$\log(h_t) = \alpha + \beta \log(h_{t-1}) + v_t \tag{24}$$

where

$$E(y_t|h_t) = \sqrt{h_t} E(e_t) = 0$$
$$E(y_t^2|h_t) = h_t E(e_t^2) = h_t \sigma_e^2$$

and

$$E(\log(h_t)|h_{t-1}) = \alpha + \beta \log(h_{t-1})$$
$$E((\log(h_t))^2|h_{t-1}) = E(v_t^2) = \tau_v^2$$

such that:

$$y_t|h_t \sim t_{df}(0, h_t\sigma_e^2) \tag{25}$$
$$\log(h_t)|h_{t-1} \sim N\left(\alpha + \beta \log(h_{t-1}), \tau_v^2\right) \tag{26}$$

We may now estimate this model using a particle filter with parameter learning in which the unknown static parameters are α,β and τ_v (which we combine in a vector θ). We assume knowledge of the degrees of freedom in $e_t \sim t_{df}$ and follow the approach of Lopes and Tsay (2011) by estimating the model with different degrees of freedom and then estimating the final posterior probability distributions by integrating over all these models. The objective is to estimate the joint probability distribution of the unobserved volatility h_t and the parameters in θ conditional on the observed returns y_t:

$$p(h_t, \theta|y_t) \tag{27}$$

For each time period, t, we sequentially produce a Monte Carlo generated series of N particles for the unobserved stochastic volatility and the unknown parameters, $\{h_{t-1}^{(i)}, \theta_{t-1}^{(i)}\}_{i=1}^N$, that approximate the density in (27) as per Lopes and Tsay (2011). [19] Each particle is then assigned a weight ($w^{(i)}$):

$$w^{(i)} \propto p(y_t|E[\log(h_{t-1}^{(i)})], m^{(i)}))$$
$$m^{(i)} = a\theta^{(i)} + (1-a)\overline{\theta}$$

where $m^{(i)}$ is a weighted average of the parameter particle $\theta^{(i)}$ and the average parameter particle value $\overline{\theta^{(i)}} = \frac{1}{N}\sum_{j=1}^N \theta^{(j)}$ with weights a and $(1-a)$.

[19] The t- subscript on θ refers to the observation period for which the set of particles were drawn. θ is still assumed to be a vector of *static* parameters.

The weights (or shrinkage constant) is set at $a = 0.95$ as in Lopes and Tsay (2011). This shrinkage constant is what allows the Liu and West (2001) filter to incorporate an artificial evolution of the parameter estimates without losing information (Lopes and Tsay, 2011).

The Liu and West (2001) algorithm then goes as follows. At each time period:

(1) Resample a new set of particles, $\{\tilde{h}_t^{(i)}, \tilde{\theta}_t^{(i)}\}_{i=1}^N$ by assigning the weights, $w^{(i)}$, to the previous set of particles, $\{h_t^{(i)}, \theta_t^{(i)}\}_{i=1}^N$.

(2a) Then propagate the resampled parameter vector $\{\tilde{\theta}_t^{(i)}\}_{i=1}^N$ to $\{\hat{\theta}_t^{(i)}\}_{i=1}^N$ via the normal distribution $N(\tilde{m}^{(i)}, V)$ where $V = (1-a^2)\sum_{j=1}^N (\theta_{t-1}^{(j)} - \overline{\theta})(\theta_{t-1}^{(j)} - \overline{\theta})'$.

(2b) Propagate new state particles $\{\tilde{h}_t^{(i)}\}_{i=1}^N$ to $\{\hat{\theta}_t^{(i)}\}_{i=1}^N$ via the density $p(h_t|\tilde{h}_{t-1}^{(j)}, \tilde{\theta}_t^{(j)})$.

(3) Resample again both state variable and parameter particles $\{(h_t, \theta_t)^{(i)}\}_{i=1}^N$ from the propagated particles $\{(\hat{h}_t, \hat{\theta}_t)^{(i)}\}_{i=1}^N$ with each particle (i) assigned the weight

$w_t^{(i)} \propto \frac{p(y_t|\hat{h}_t^{(i)}, \hat{\theta}_t^{(i)})}{p(y_t|E(\tilde{h}_{t-1}^{(i)}), \tilde{m}^{(i)})}$.

We then store the posterior distribution at each observation point, and repeat the algorithm at the following observation point. The output once the algorithm is run through the entire data set gives us a filtered estimate of the underlying latent volatility process. In this exercise the estimate is only filtered and not smoothed, in the sense that only past observations were included in the information set that the probability distribution is conditional upon.[20]

6. The structure of volatility spillovers

As discussed in the introduction, the choice of proxy for volatility may have a strong influence on the estimated spillover index. In particular, we are interested in the fact that squared returns as a measure of volatility is plagued by high *noise-to-signal* ratios, a ratio that increases with increased volatility (Andersen et al., 2006). The stochastic volatility model, on the other hand, removes the noise and produces an estimate of the *underlying* latent volatility process. In this section we will therefore reestimate the volatility spillover index using an estimate of the individual stochastic volatility for each currency.

The calculation of a separate spillover index that uses stochastic volatility models enables us to determine whether the changes in the spillover indexes for total observed volatility (that use the squared returns measure) may be

[20]For further details on the filtering algorithm see Lopes and Tsay (2011) and Liu and West (2001).

attributed to the underlying (more persistent) element of volatility, or whether these changes are due to volatility shocks. This is an area of potential interest, since the period that preceded the recent global financial crisis was characterized by levels of excessive debt,[21] and other relatively 'slow moving' events (such as those that may have resulted in slight changes to the structure of an economy). Therefore, it would be of interest to determine whether the changes to the measure of underlying volatility, that is described by the stochastic volatility model, have impacted on the spillover in volatility from one country to another, or whether the spillover index is largely attributed to volatility shocks to these currencies.

Figure 8 plots the stochastic volatility estimates with the squared returns of the same series. Stochastic volatility estimates and squared returns for the remaining currencies are reported in Appendix C, figures C.23 - E.41. The posterior estimates of the parameters, θ, in the model are reported in Appendix E.

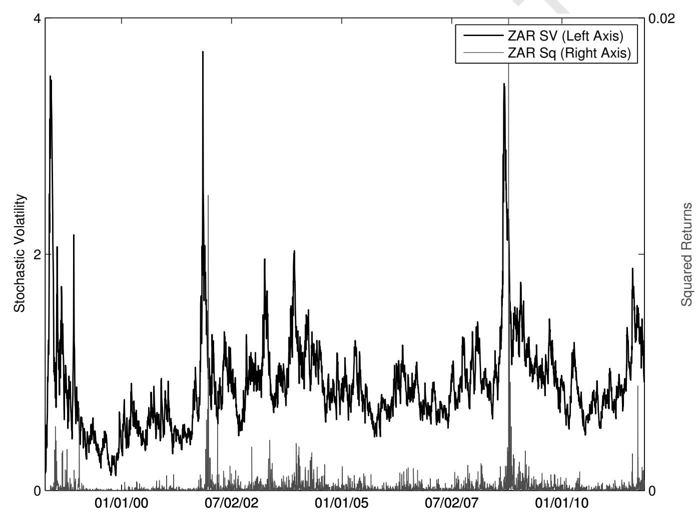

Figure 8: Stochastic volatility (black line, left axis) and squared returns (grey line, right axis)

As expected, it appears that the squared returns exaggerate the effects of high volatility incidents compared to the stochastic volatility estimates which are slightly smoother (the extreme variation in the squared returns on the South African rand is less visible due to the scale of the y-axis which is adapted to capture the outliers in volatility in 2008 and 2002). Now, after reducing the *noise-to-signal* ratio of our volatility estimate, we may proceed to reestimate the spillover index for currency volatility. In the following, we will refer to

[21]See, Reinhart and Rogoff (2009), "This time is different".

the index based on the stochastic volatility proxy as the *underlying volatility spillover index*. The index based on the squared returns proxy is simply referred to as the *squared returns spillover index*. Comparing the two indexes should reveal whether it is the noise or the *underlying* volatility that accounts for changes in the volatility spillovers.

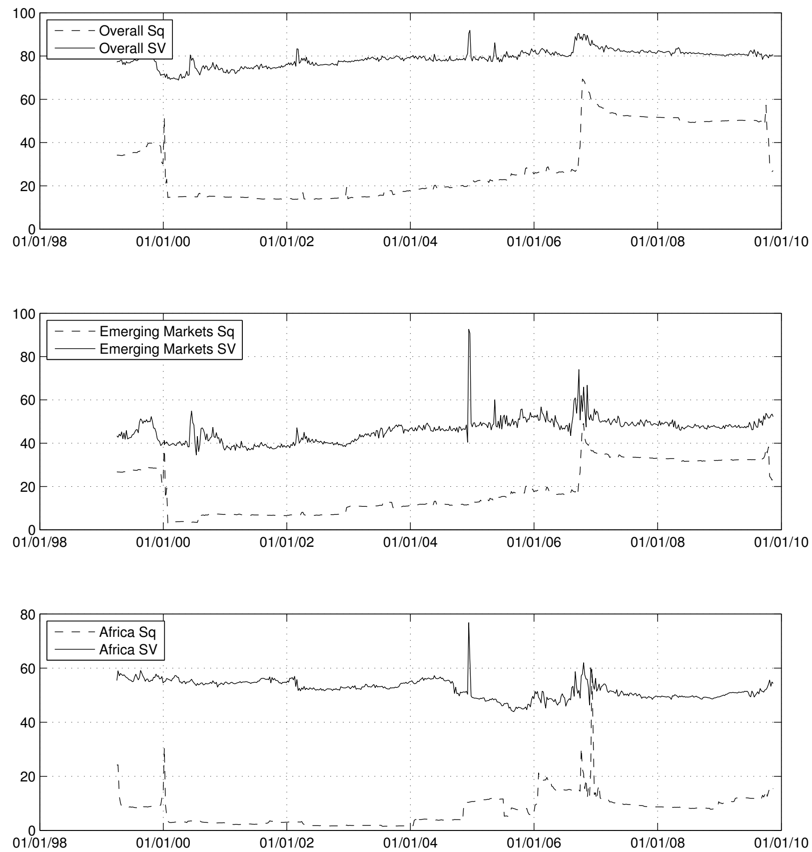

Figure 9: Overall, Emerging Market and Africa spillover index in volatility, based on stochastic volatility estimates

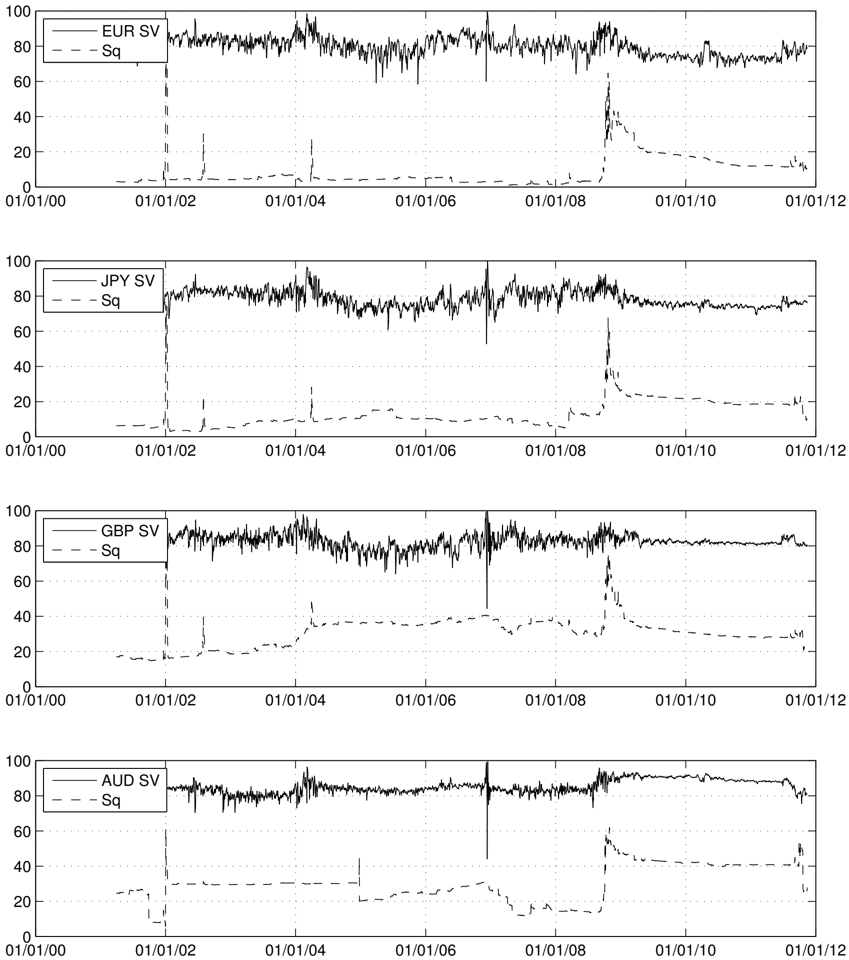

Figure 10: Individual spillover index in volatility, based on stochastic volatility estimates

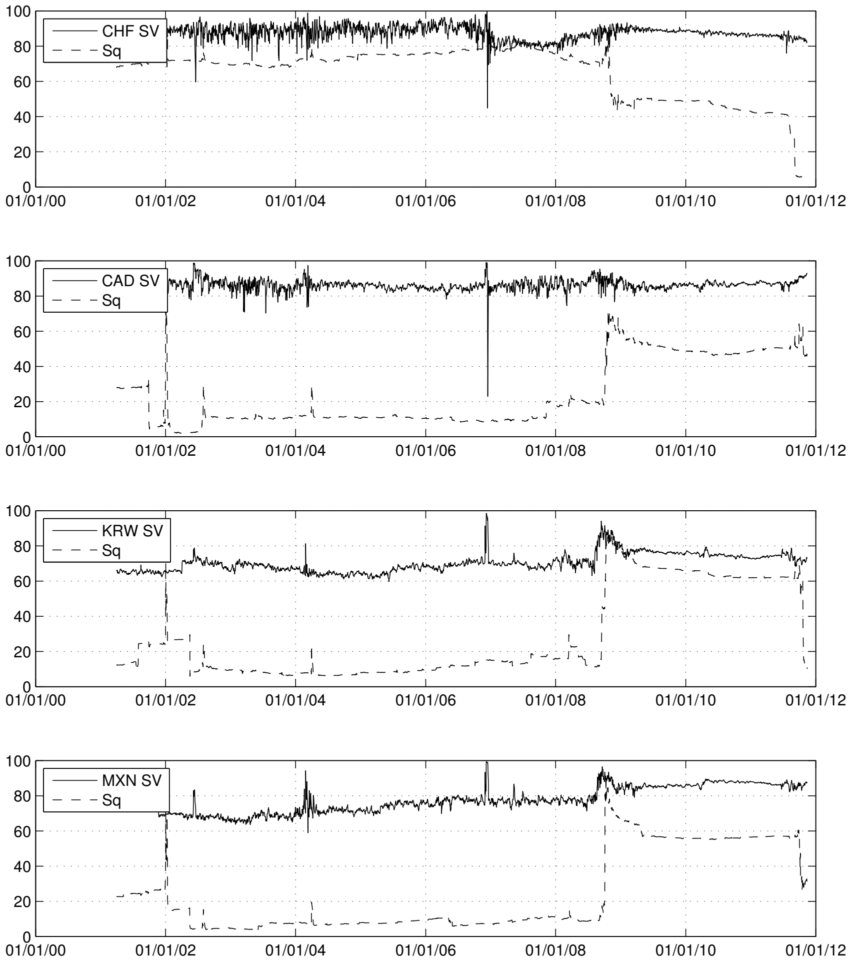

Figure 11: Individual spillover index in volatility, based on stochastic volatility estimates

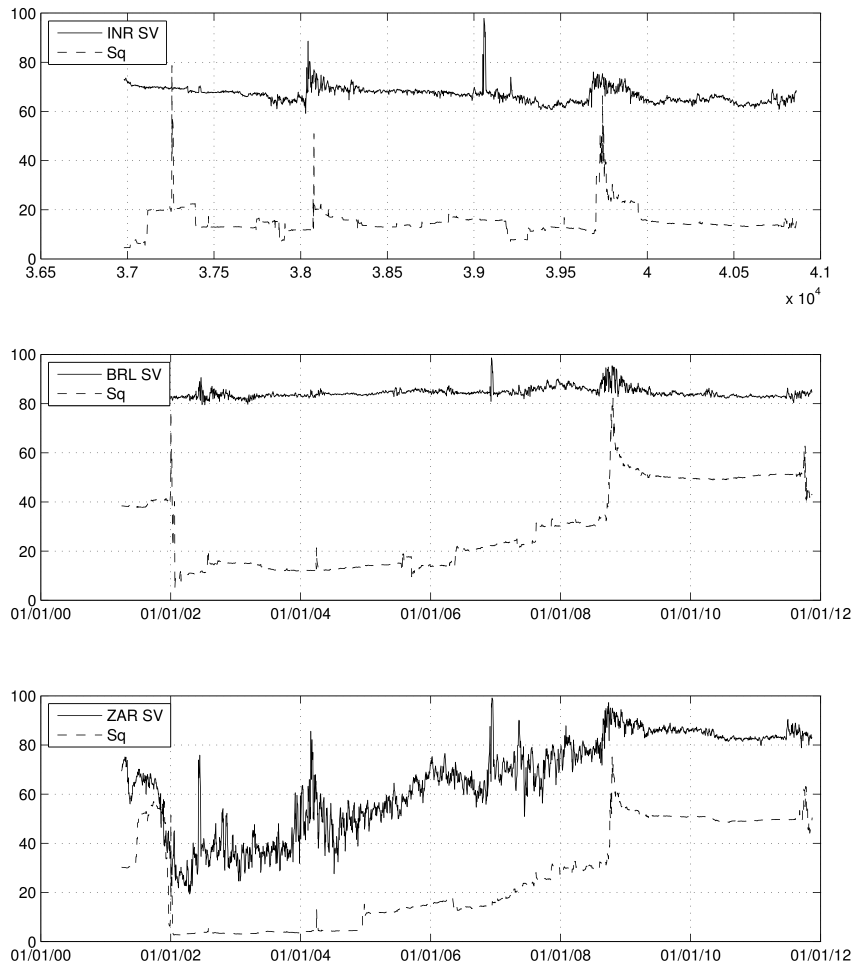

Figure 12: Individual spillover index in volatility, based on stochastic volatility estimates

The resulting *underlying volatility spillover indexes* display some rather unexpected characteristics. Firstly, they appear less smooth than the *squared returns spillover index*. Secondly, the *underlying volatility spillover index* does not respond strongly to the financial crisis, and does not display a clear upwards trend. And lastly, this index is estimated to be very high for most currencies: Spillovers consistently explain approximately 80% of the variance in the *underlying* volatility process, but only 15% - 30% of variance in the volatility *noise* during normal times and between 50% and 80% during times of financial turmoil. The *underlying volatility spillover index* is also higher than what Diebold and Yilmaz (2009) found for volatility spillovers in equity markets (40-80%). The following paragraphs will discuss what we may learn from these characteristics.

Inspection of figure 9 reveals that both spillover estimates react to the financial crisis of 2008 and both also appear to have been high in the aftermath of the dot-com bubble that burst in 2001. The *squared returns spillover index* displays a positive trend between these events, at a slightly higher growth rate than the *underlying volatility spillover index*. At the very end of the sample, the *squared returns index* dips back down to levels seen prior to the crisis, while the *underlying volatility spillover index* shows no strong reduction. The responsiveness of the *squared returns index* shows that the variance of noise is dominated by global shocks and cross-country linkages during financial crises, perhaps a symptom of contagion. While the *underlying* volatility process shows no symptoms of contagion and will spill over across currencies during crises at approximately the same rate as during normal times. Generally, it appears that cross-country linkages are weaker and more variable in exchange rate noise than in underlying volatility.

Figure 12 illustrates that the *underlying volatility spillover index* for the South African rand has displayed a clear positive trend, similar to the spillover index for volatility noise. Notably, no other currencies display this positive trend in *underlying volatility spillovers*, and we suspect that it is purely the South African rand that causes the overall *underlying volatility spillover index* to have a slightly positive trend. The rand index is also different from the rest in that the *underlying volatility spillovers* have been relatively low (30%) at times. The drop down to this level happened at the end of 2001 and the beginning of 2002, and another albeit slower reduction in the spillover index happened in 2006. Notably both these occurrences happened at times that has been identified as currency crises in the South African rand (Knedlik and Scheufele, 2008; Knedlik, 2006). This is an important finding, as it gives evidence that the rand crisis of 2001-2002 not only made the volatility noise act independently of other currencies, but also the *underlying spillover index* was strongly affected. And it took approximately 6-8 years before the spillovers in the *underlying* volatility returned to past levels. Only at the onset of the global financial crisis did the rand's *underlying volatility spillover index* reach the level observed for other currencies. It appears that the rand was headed to this level at a slow pace possibly in step with increased financial integration, but that the global financial crisis of 2008 sped up this convergence. This can be seen by the slightly stronger reaction of the rand spillover index to the financial crisis, compared to the other currencies.

Another currency in our sample that provides a highly interesting case study is the Swiss Franc (CHF). It appears that the announcement on the 6th of September 2011 that the Swiss National Bank (SNB) would put a ceiling on the CHF/EUR exchange rate has greatly affected its *squared returns spillover index*. Spillovers in exchange rate noise dropped by 25 percentage points after this announcement and are now lower than at any other point in our sample. Interestingly, there is also a very slight dip in the spillovers of *underlying* volatility. Due to the smoothness of underlying volatility, it will only respond to a more permanent intervention (or structural change) in the exchange rate. In the SNB's latest Quarterly Bulletin, it is confirmed that "The Swiss National Bank (SNB) will continue to enforce the minimum exchange rate of CHF 1.20 per euro with the utmost determination" (SNB, 2011). This will therefore provide a highly interesting case to follow into the near future as the ceiling remains in place. It would be surprising if the *underlying* volatility spillovers do not continue to decrease as the central bank keeps intervening at such a considerable extent.

6.0.1. Implications for practitioners

What do we learn from the fact that underlying volatility is characterized by consistently high spillovers, while spillover in currency noise is low in calm markets and high during crisis? Firstly, in the longer term it appears that underlying currency volatility is barely related to shocks to the respective currency but almost completely driven by spillovers from shocks to other currencies. However, the variance of day to day noise is in the short term largely controlled by shocks to the respective currency unless the market is suffering from global financial stress. At the peak of crises, this noise may be driven as much by spillovers as the *underlying* volatility.

This knowledge may be very useful for risk management and option pricing. It says that when forecasting currency risk, one should worry more about global shocks rather than domestic country specific shocks. However, when assessing the risk of large price moves in the very near future, one should be more concerned by shocks to the respective economy. The exception is a global financial crisis which appears to trump domestic shocks also with respect to short term noise. It is beyond the scope of this study to investigate the potential of our findings for improved volatility forecasts. But we see scope to improve the accuracy of the *variance* of a currency volatility forecast. That is, incorporating information about shocks to other currencies and the *underlying volatility spillover index* may improve the accuracy of the *significance interval* of the forecast. This is because our spillover index adds information about the sources of this forecast variance. In other words, we may not improve accuracy of the forecast itself, but rather improve our knowledge about *the accuracy of the forecast.*

The findings are also of interest in terms of hedging against volatility risk related to a particular market segment, say emerging markets. This would ideally be done by using a currency where the spillovers from *emerging market* currencies are consistently high, while domestic shocks should have, ideally, no effects. That is, one would want the currency to be consistently dominated by the market against which one hedges (assuming that one can short this currency). The spillover index can easily be estimated to include only emerging market shocks on a currency, and one would be able to use this index to choose the most appropriate currency for the hedge. The downside is of course that the hedge would only be useful in the long run, as the short run volatility noise is largely driven by country specific shocks. Another concern would be an unexpected persistent shock to the economy as this may also reduce the *underlying* spillover index as we saw with the Swiss franc and South African rand.

To summarize, this section have highlighted how the choice of volatility proxy affects the estimated spillover effects. Given the striking similarities between our *squared returns spillover index* and Diebold and Yilmaz (2009)'s range based *volatility spillover index* in world equity markets, it appears that a similar exercise related to their study would be of interest. It is argued that future research on volatility spillovers should differentiate between spillovers in short term price noise and the longer term underlying volatility. While spillovers in underlying volatility tend to be much more dominant than spillovers in the price noise they also tend to react more moderately to financial events. This was apparent during the financial crisis of 2008 when the spillovers in price noise reached the same high levels as spillovers in underlying volatility. Large changes in underlying volatility spillovers would be caused by persistent shocks to the specific currency as this would over time pull the *underlying volatility spillover index* to a lower level.

7. Concluding remarks

The paper has provided a rigorous investigation of spillover effects in the foreign exchange market, with a special focus on emerging market currencies and the South African rand. The framework was based upon the spillover index as suggested by Diebold and Yilmaz (2009) and a particle filter following the Liu and West algorithm. A spillover index for returns and volatility was estimated for regions and individual currencies. Two different proxies for volatility were used; (1) squared returns and (2) a stochastic volatility estimate. It is argued that the high *noise-to-signal* ratio of squared returns makes this proxy more like a measure of short term price noise, while the stochastic volatility estimate reflects the long term underlying volatility process. These characteristics should be reflected in the spillover index and accordingly we estimate a spillover index for each of the measures.

The resulting spillover index for returns and squared returns (price noise) are strikingly similar to those found by Diebold and Yilmaz (2009) for equity markets. It is found that return spillovers are characterized by a positive and rather smooth trend with no strong reaction to financial crises or other occurrences of financial stress. The spillover index for squared returns, on the other hand, display more abrupt changes in response to global financial events such as the bursting dot-com bubble of 2001 and the financial crisis of 2008.

The spillover index for the underlying volatility process is consistently very high, around 80% for most currencies, and displays no apparent change in the trend. A moderate reaction to the global financial crisis is indicated, much like the reaction of return spillovers. However, the spillover index for underlying volatility in the South African rand has behaved more similarly to its index for squared returns, with a positive trend and more abrupt movements. It appears that the underlying volatility index responded strongly to South African specific events, such as the currency crises of 2001 and 2006. As one would expect, a country specific crisis will reduce the spillover index for the respective currency, as it is country specific shocks that influence the volatility. This is of course in contrast to the event of a global financial crisis where the respective currency is driven more by global shocks or shocks to other currencies. It should also be noted that in the aftermath of the financial crisis of 2008, the spillovers in underlying volatility of the South African rand has reached the level of other currencies and seems to be stable at this level. It is found that the returns and squared returns spillover indexes for South Africa both behave similarly to those of other major emerging markets such as the Korean Won, Mexican Peso and Brazilian Real. These emerging market currencies were all characterized by a considerable increase in return spillovers, from less than 20% in 2002 to more than 60% in 2011. The same currencies saw the spillover index for squared returns move from less than 20% prior to the global financial crisis to between 70% and 100% at the peak of the crisis. All indexes, for returns and both volatility measures, appear to have stabilized in the aftermath of the crisis although some reductions in spillovers appear to happen at the very end of the sample.

Lastly, it is argued that future research on volatility spillovers, not only in foreign currency markets but also equities and other asset classes, should incorporate a study of the discrepancies between spillovers in long term underlying volatility and the short term price noise. The results of this paper provide evidence that the spillover effects of the two types of volatility are likely to behave differently over time and a study of their behavior provides a highly interesting subject for further analysis.

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Appendix A. Alternative Cholesky ordering

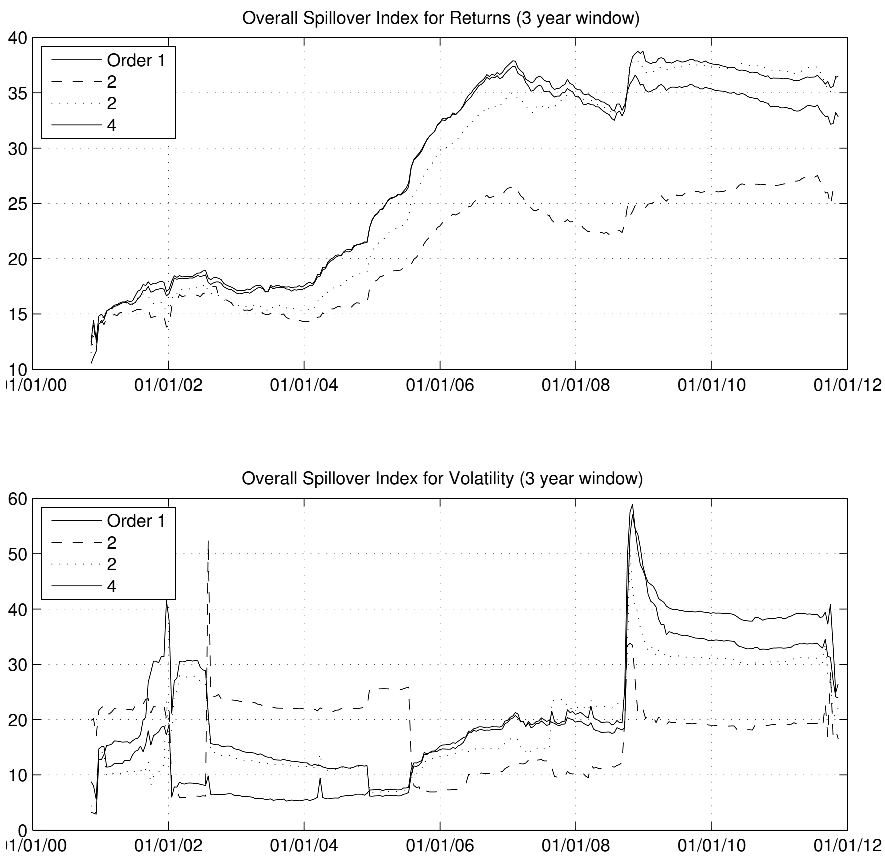


Figure A.13: Spillover index 720 day rolling window with alternative ordering

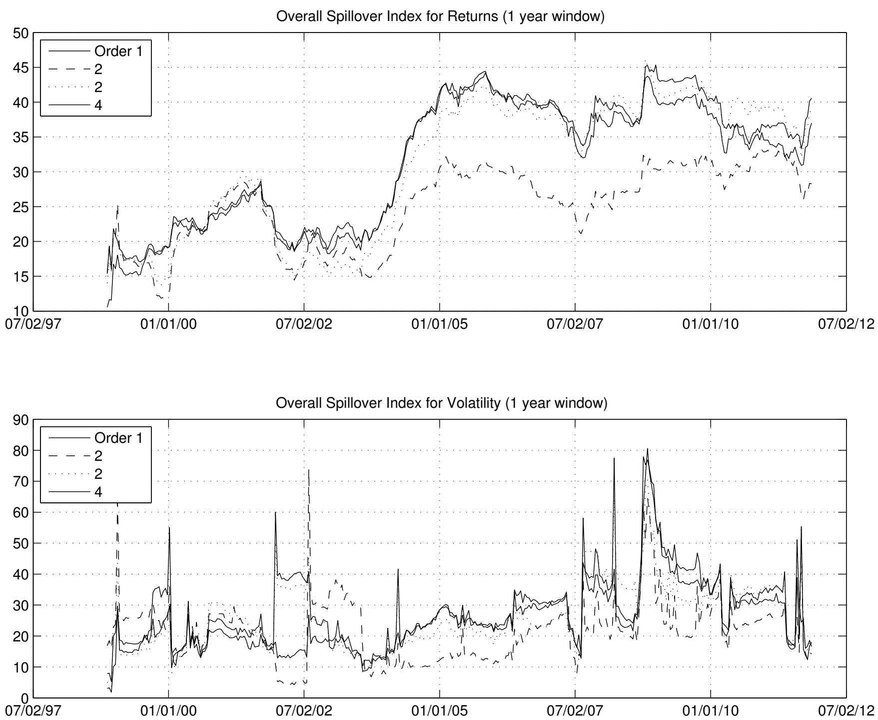
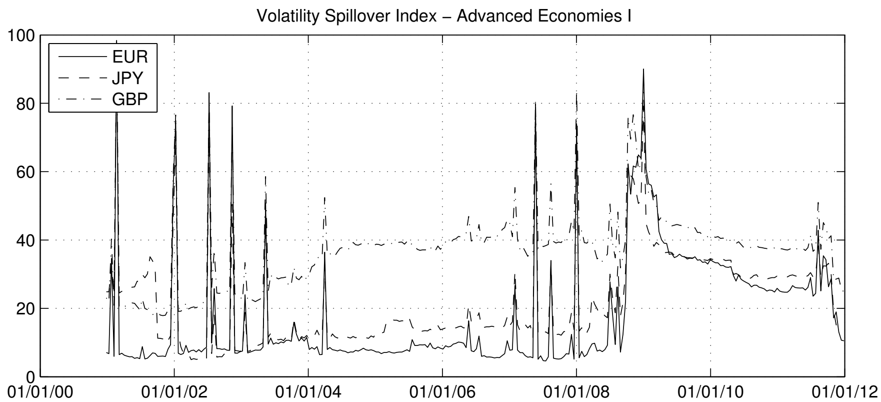


Figure A.14: Spillover index 260 day rolling window with alternative ordering

Appendix B. Individual currency spillover index



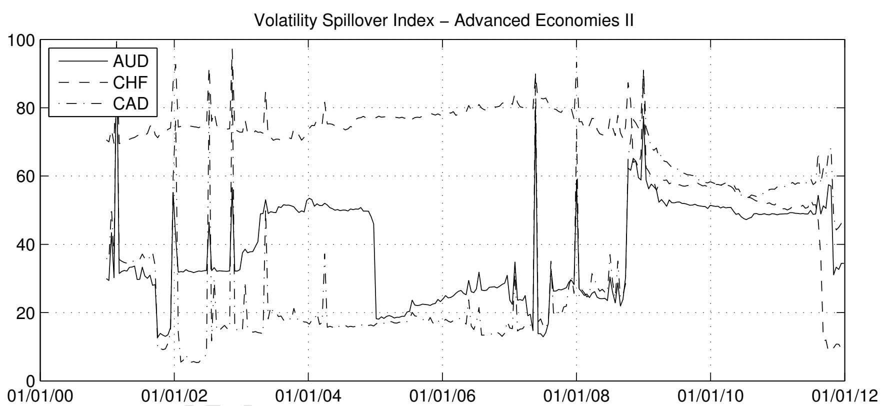


Figure B.15: Volatility spillover index for individual countries. Rolling 720 day window.

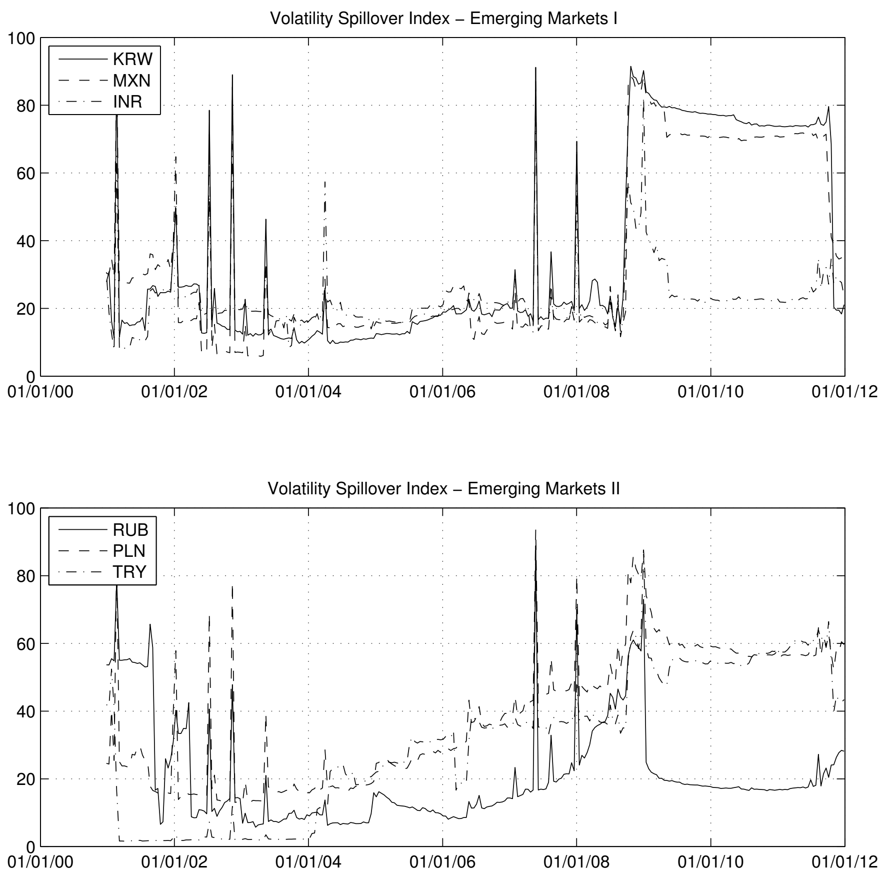
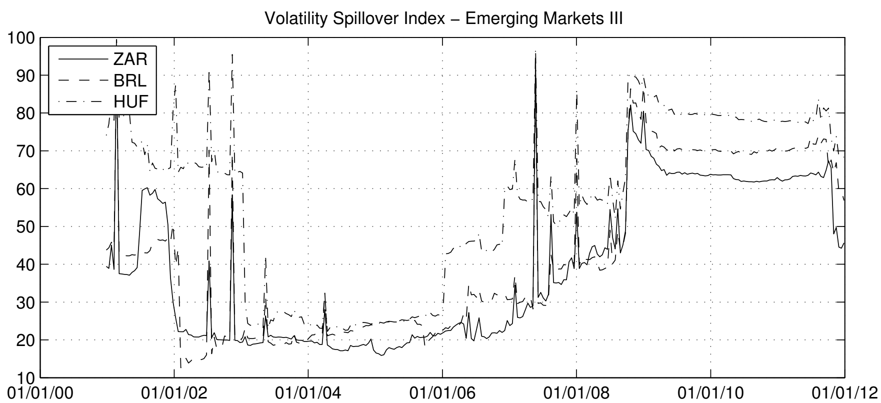


Figure B.16: Volatility spillover index for individual countries. Rolling 720 day window.



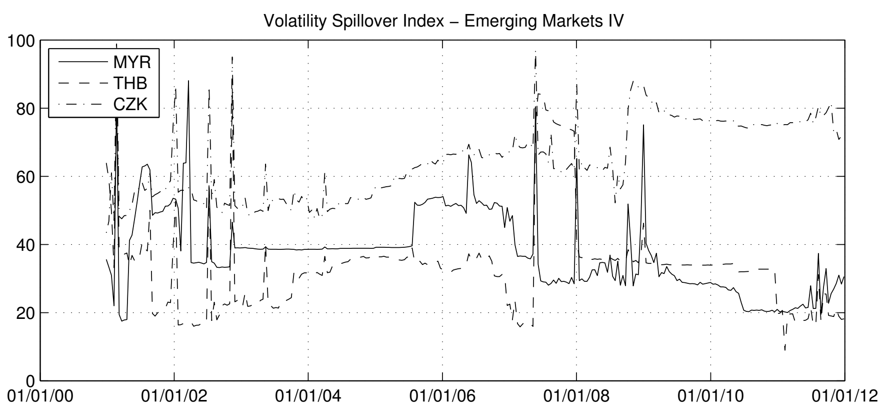


Figure B.17: Volatility spillover index for individual countries. Rolling 720 day window.

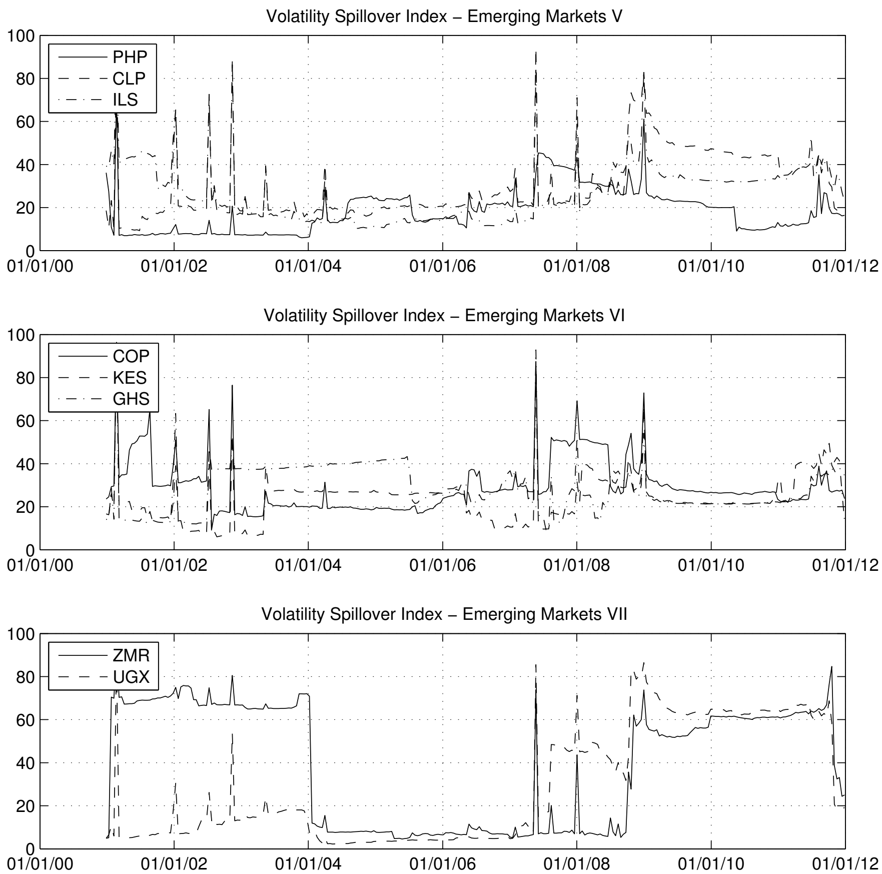


Figure B.18: Volatility spillover index for individual countries. Rolling 720 day window.

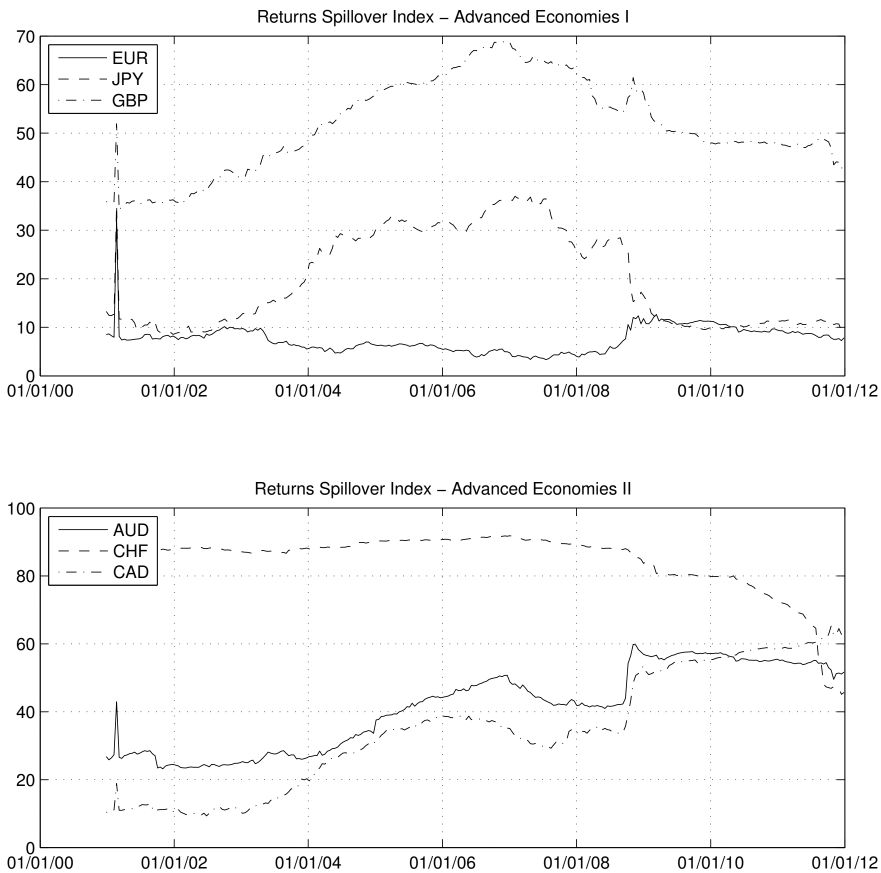
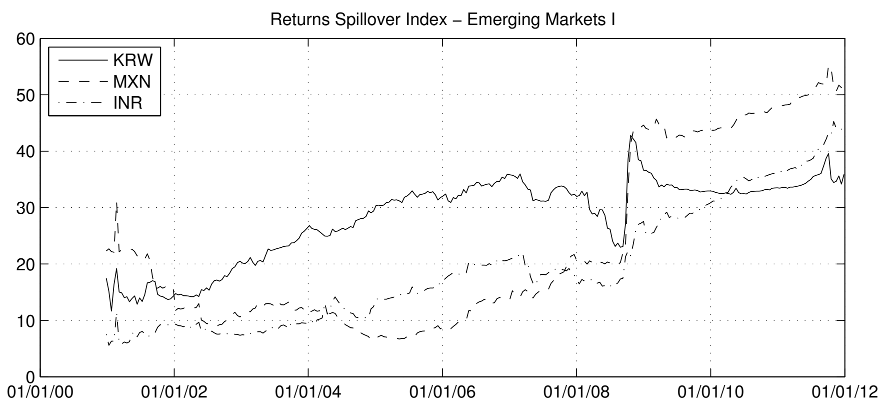


Figure B.19: Returns spillover index for individual countries. Rolling 720 day window.



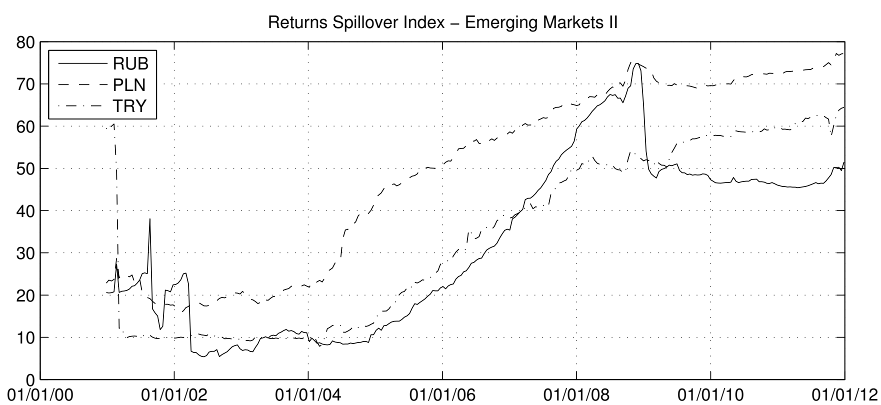
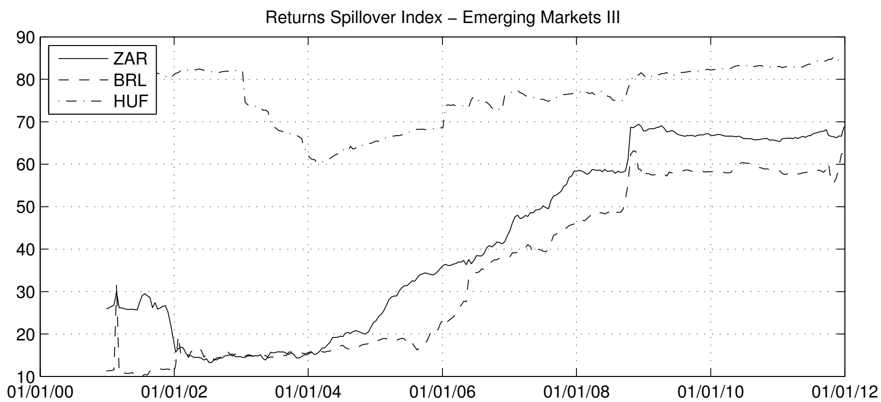


Figure B.20: Returns spillover index for individual countries. Rolling 720 day window.



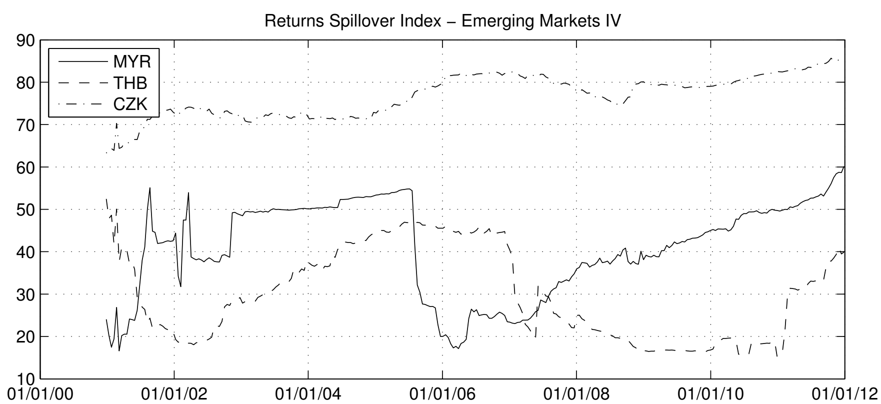


Figure B.21: Returns spillover index for individual countries. Rolling 720 day window.

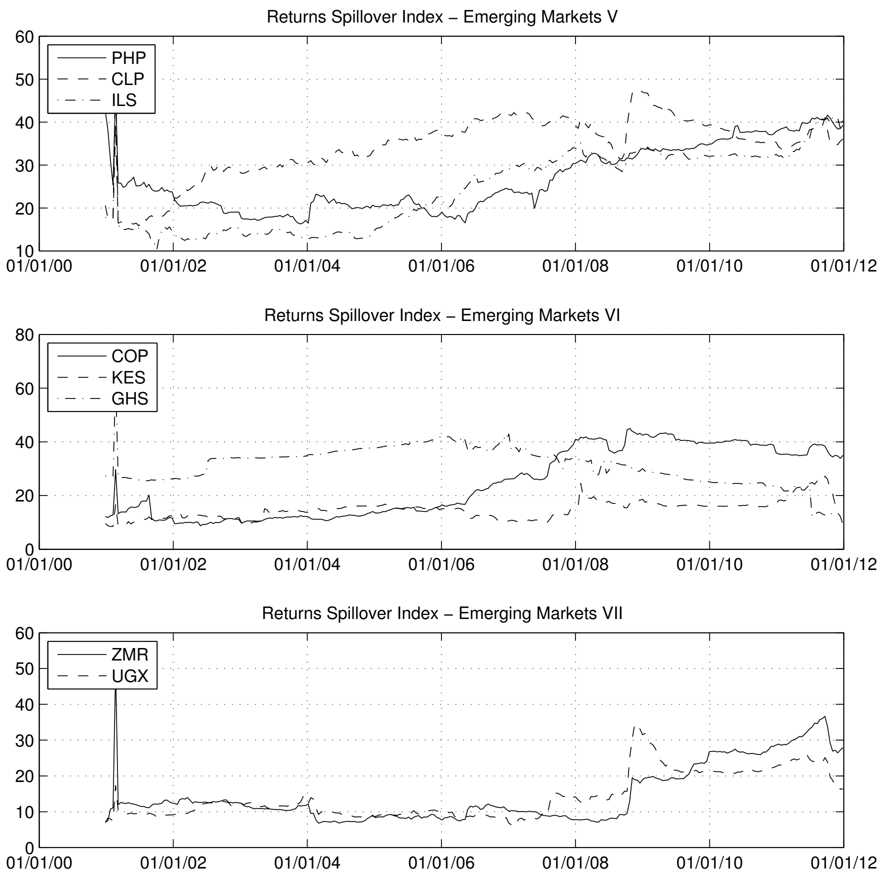


Figure B.22: Returns spillover index for individual countries. Rolling 720 day window.

Appendix C. Stochastic volatility estimates

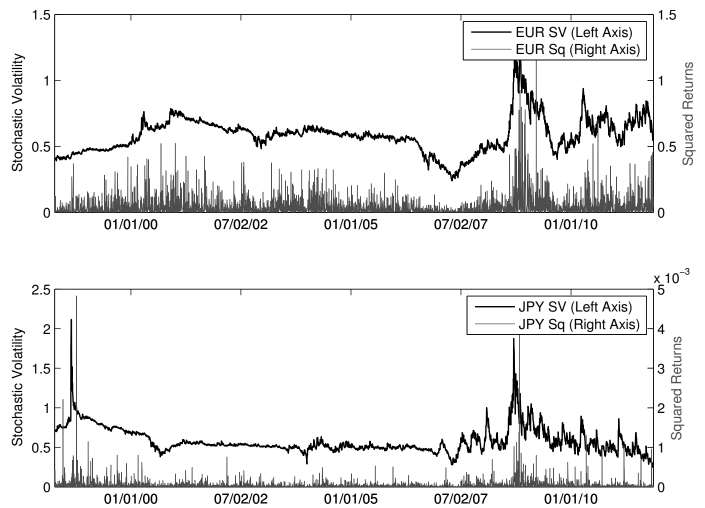


Figure C.23: Stochastic volatility (black line, left axis) and squared returns (grey line, right axis)

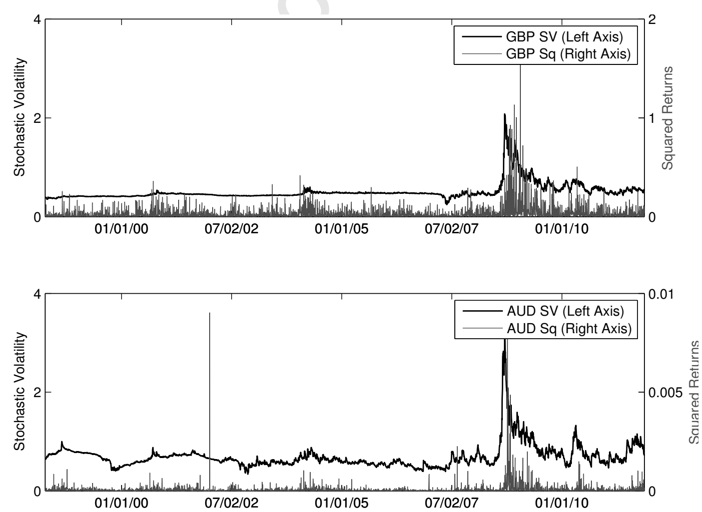


Figure C.24: Stochastic volatility (black line, left axis) and squared returns (grey line, right axis)

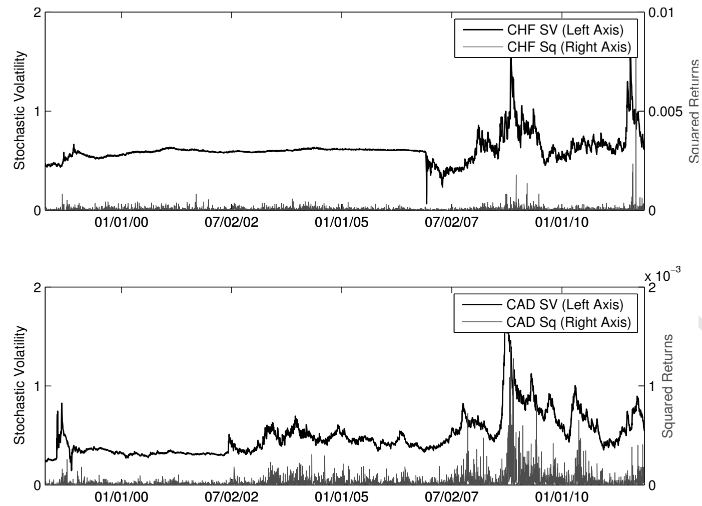


Figure C.25: Stochastic volatility (black line, left axis) and squared returns (grey line, right axis)

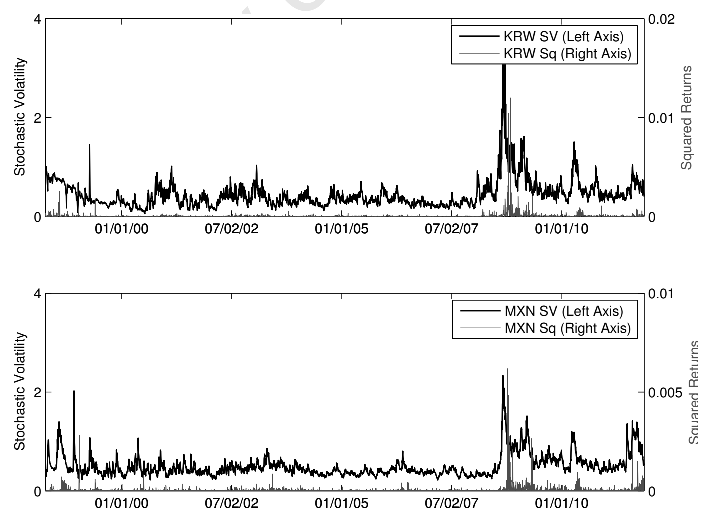


Figure C.26: Stochastic volatility (black line, left axis) and squared returns (grey line, right axis)

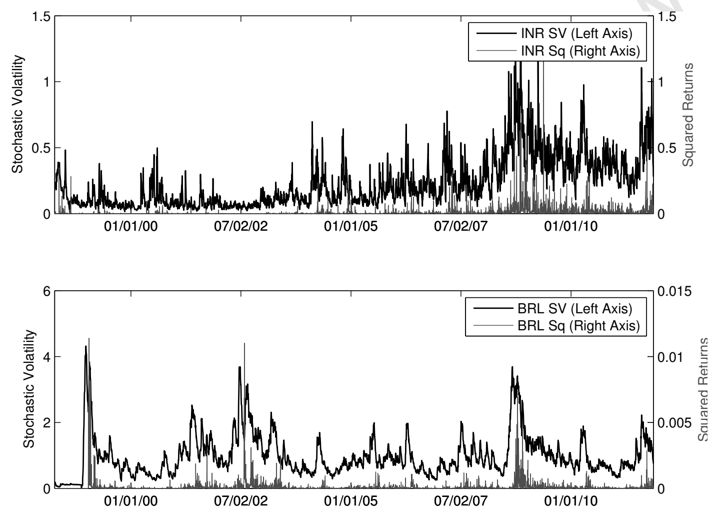


Figure C.27: Stochastic volatility (black line, left axis) and squared returns (grey line, right axis)

Appendix D. Generalized and ordered impulse response functions

The following illustrates how the estimated spillover index will differ when based on Generalized impulse response functions (GIRF) as opposed to ordered impulse response functions (OIRF). The derivations are based on the statement of Kim (2009) that the GIRF from a shock to variable i equals to the OIRF from this shock when variable i is on top of the Cholesky ordering.[22]

We illustrate the difference for a simple two-variable first-order VAR model, beginning with the primitive (structural) form.

Primitive form:

$$y_t = b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \tag{D.1}$$

$$z_t = b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \tag{D.2}$$

Moving Average:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \sum_{i=0}^{\infty} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^i \begin{pmatrix} e_{1,t-i} \\ e_{2,t-i} \end{pmatrix} \tag{D.3}$$

Where:

$$\begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix} = \frac{1}{1 - b_{12}b_{21}} \begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} \tag{D.4}$$

We insert (4) in (3):

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \frac{1}{1 - b_{12}b_{21}} \sum_{i=0}^{\infty} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^i \begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{pmatrix} \tag{D.5}$$

We may find the GIRFs from a shock to $\varepsilon_{y,t}$ by setting $b_{21} = 0$ (i.e. $\varepsilon_{y,t}$ has a contemporaneous effect on y_t, but not on z_t).

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix}_{GIRF,\varepsilon_{y,t}} = \sum_{i=0}^{\infty} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^i \begin{pmatrix} 1 & b_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{pmatrix} \tag{D.6}$$

Similarly, we find the GIRFs from a shock to $\varepsilon_{z,t}$ by setting $b_{12} = 0$ (i.e. $\varepsilon_{z,t}$ has a contemporaneous effect on z_t, but not on y_t).

[22] By "top" ordering we mean that shocks to variable i are assumed to have no contemporaneous effects on any other variables than i, but the shock can affect all other variables at a lag.

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix}_{GIRF,\varepsilon_{z,t}} = \sum_{i=0}^{\infty} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^i \begin{pmatrix} 1 & 0 \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{pmatrix} \tag{D.7}$$

Adding out the infinite sum:

$$\begin{aligned} \begin{pmatrix} y_t \\ z_t \end{pmatrix}_{GIRF,\varepsilon_{y,t}} &= \begin{pmatrix} 1 & 0 \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} \\ &+ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-1} \\ \varepsilon_{z,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} a_{11}^2 + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{21}a_{11} + a_{22}a_{21} & a_{21}a_{12} + a_{22}^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-2} \\ \varepsilon_{z,t-2} \end{pmatrix} \\ &+ \dots \end{aligned}$$

Which again equals to (after multiplying the coefficient matrices):

$$\begin{aligned} \begin{pmatrix} y_t \\ z_t \end{pmatrix}_{GIRF,\varepsilon_{y,t}} &= \begin{pmatrix} 1 & 0 \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} \\ &+ \begin{pmatrix} a_{11} + a_{12}b_{12} & a_{12} \\ a_{21} + a_{22}b_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-1} \\ \varepsilon_{z,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} a_{11}^2 + a_{12}a_{21} + (a_{11}a_{12} + a_{12}a_{22})b_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{21}a_{11} + a_{22}a_{21} + (a_{21}a_{12} + a_{22}^2)b_{21} & a_{21}a_{12} + a_{22}^2 \end{pmatrix} \begin{pmatrix} \varepsilon_{y,t-2} \\ \varepsilon_{z,t-2} \end{pmatrix} \\ &+ \dots \end{aligned}$$

The GIRF of y_t from a shock to $\varepsilon_{z,t}$, holding $\varepsilon_{y,t} = 0$, is derived from:

$$y_t = 0 + a_{12}\varepsilon_{z,t-1} + (a_{11}a_{12} + a_{12}a_{22})\varepsilon_{z,t-2} + \dots \tag{D.8}$$

Which gives the GIRF: $\begin{pmatrix} 0 \\ a_{12} \\ a_{11}a_{12} + a_{12}a_{22} \\ \dots \end{pmatrix}$

This can be compared to the OIRF from a Cholesky ordering where $\varepsilon_{z,t}$ is allowed to have contemporaneous shocks on y_t:

$$y_t = b_{12}\varepsilon_{z,t} + (a_{11}b_{12} + a_{12})\varepsilon_{z,t-1}((a_{11}^2 + a_{12}a_{21})b_{12} + a_{11}a_{12} + a_{12}a_{22})\varepsilon_{z,t-2} + \dots$$

which gives the OIRF: $\begin{pmatrix} b_{12} \\ a_{11}b_{12} + a_{12} \\ (a_{11}^2 + a_{12}a_{21})b_{12} + a_{11}a_{12} + a_{12}a_{22} \\ \cdots \end{pmatrix}$

The impulse responses of variable i from shocks to variable j are thus lower than they would have been under a cholesky ordering, assuming positive a_{ij} and b_{ij}. This was demonstrated in the example above where the two impulse response functions of y_t from a shock to $\varepsilon_{z,t}$ were found to be :

$$\text{OIRF: } \begin{pmatrix} b_{12} \\ a_{11}b_{12} + a_{12} \\ (a_{11}^2 + a_{0,12}a_{21})b_{12} + a_{11}a_{12} + a_{12}a_{22} \\ \cdots \end{pmatrix} \text{ GIRF: } \begin{pmatrix} 0 \\ a_{12} \\ a_{11}a_{12} + a_{12}a_{22} \\ \cdots \end{pmatrix}$$

Hence, if at least one of a_{ij} and b_{ij} is non-zero, then the GIRF will differ from the OIRF. Assuming positive a_{ij} and b_{ij}, the implications for the spillover index is that the share of the variance in y_t that is explained by shocks to other currencies is likely to be lower when estimated from GIRFs rather than OIRFs. This gives a lower spillover index. In terms of "own shocks", the OIRF and GIRF of variable i should respond similarly to shocks to $\varepsilon_{i,t}$.

Appendix E. Stochastic volatility parameter estimates

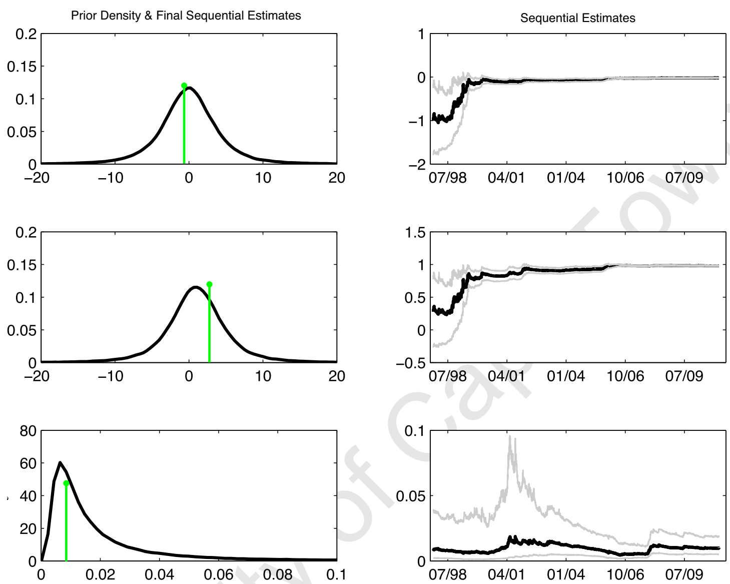


Figure E.28: Parameter estimates from the stochastic volatility model for the EU euro (EUR)

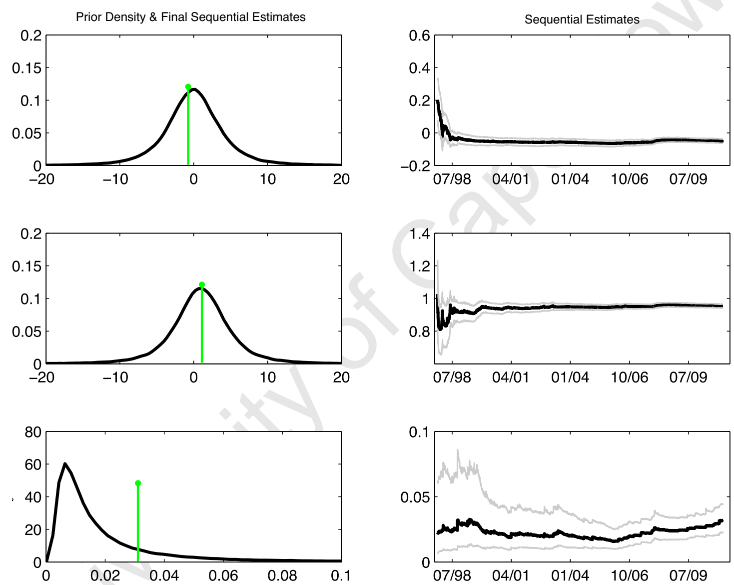


Figure E.29: Parameter estimates from the stochastic volatility model for the Japanese yen (JPY)

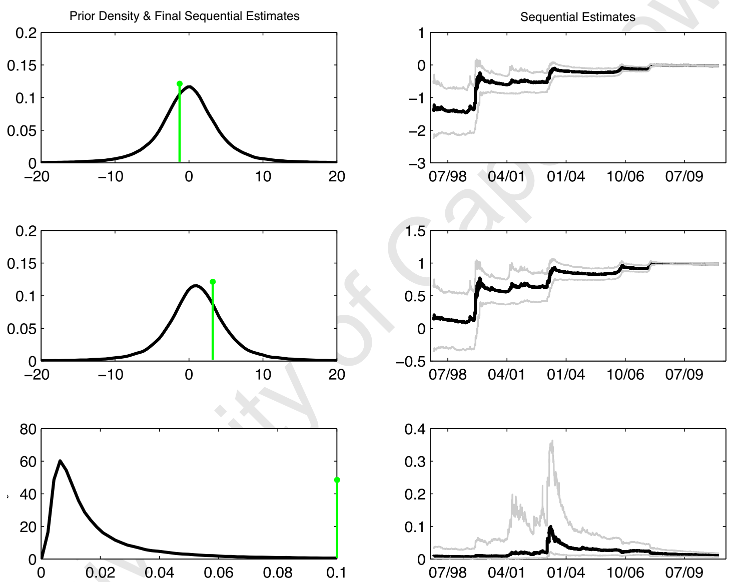


Figure E.30: Parameter estimates from the stochastic volatility model for the British pound (GBP)

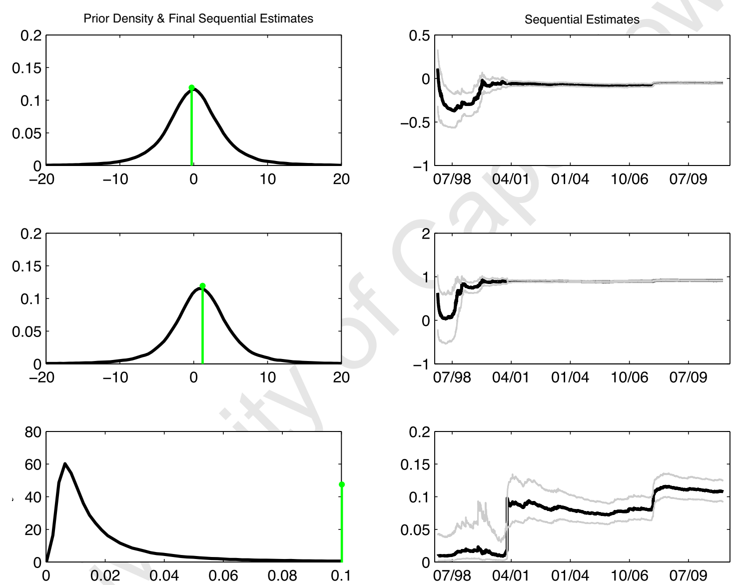


Figure E.31: Parameter estimates from the stochastic volatility model for the Australian dollar (AUD))

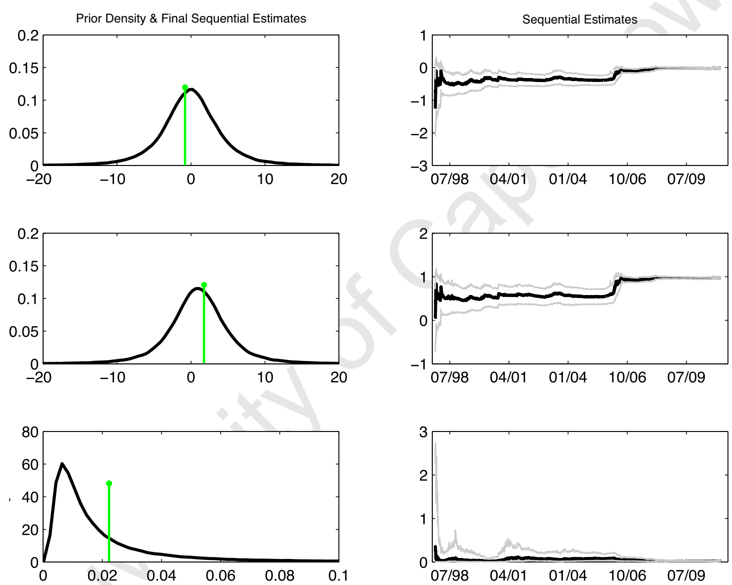


Figure E.32: Parameter estimates from the stochastic volatility model for the Swiss franc (CHF)

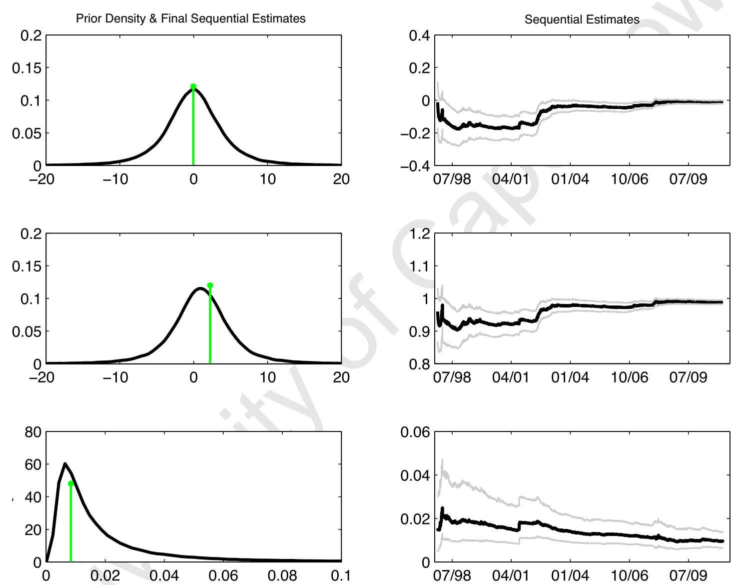


Figure E.33: Parameter estimates from the stochastic volatility model for the Canadian dollar (CAD)

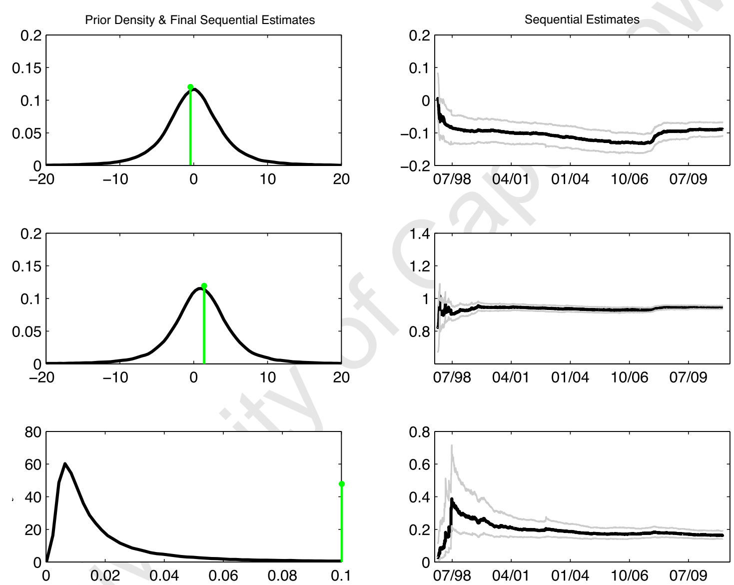


Figure E.34: Parameter estimates from the stochastic volatility model for the Korean won (KRW)

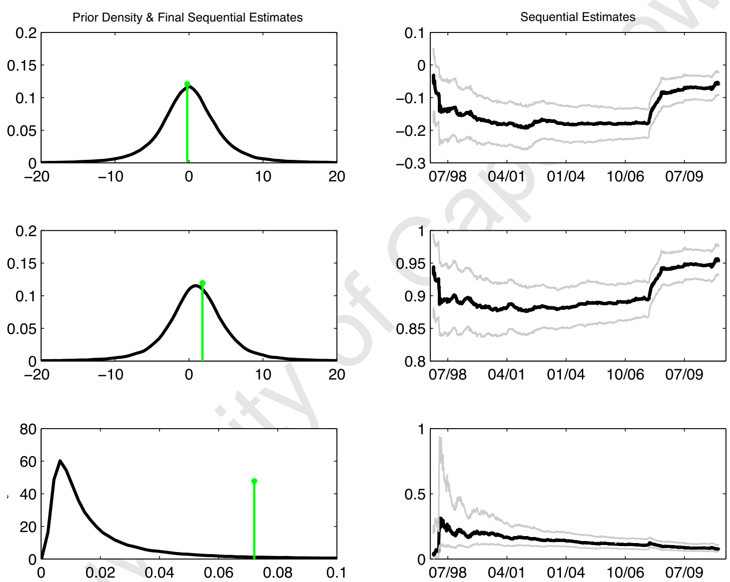


Figure E.35: Parameter estimates from the stochastic volatility model for the Mexican peso (MXN)

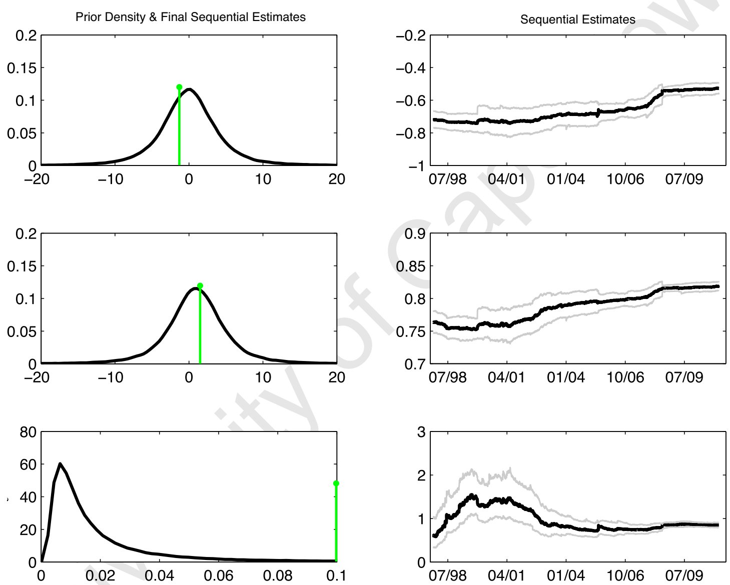


Figure E.36: Parameter estimates from the stochastic volatility model for the Indian rupee (INR)

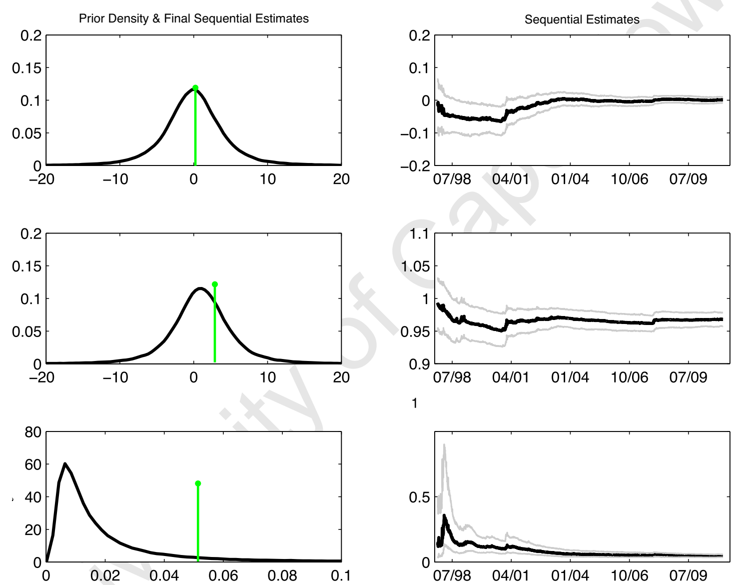


Figure E.37: Parameter estimates from the stochastic volatility model for the South African rand (ZAR)

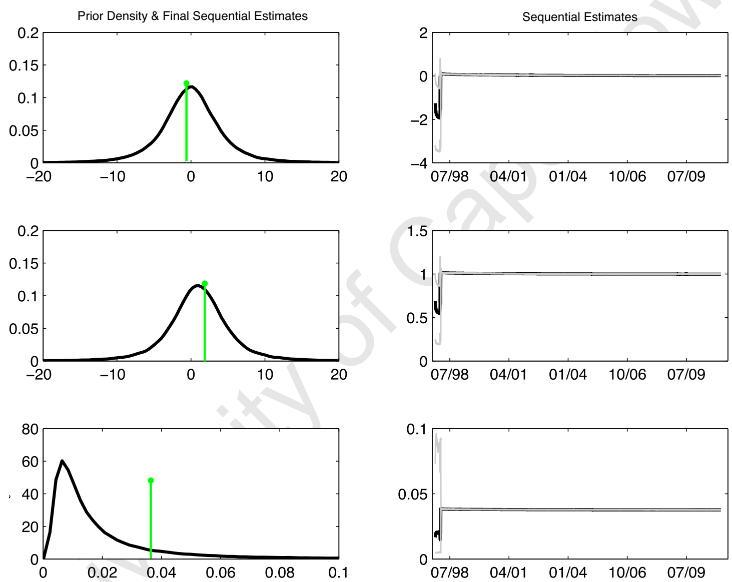


Figure E.38: Parameter estimates from the stochastic volatility model for the Brazilian real (BRL)

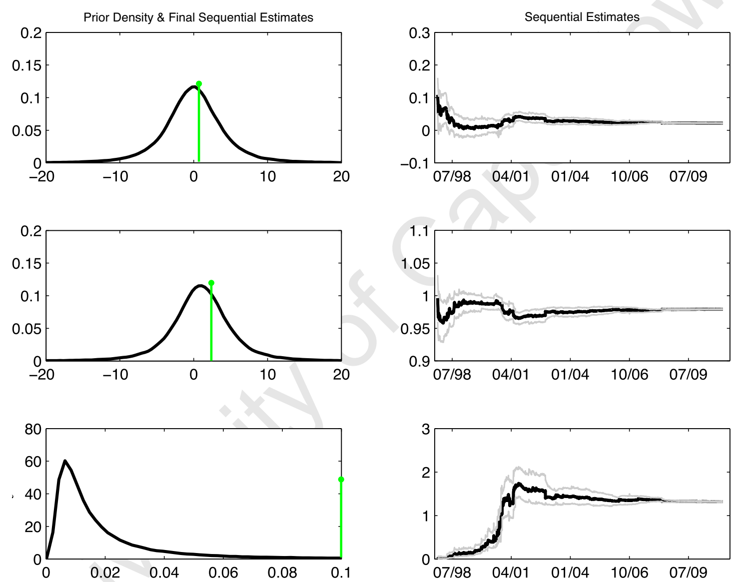


Figure E.39: Parameter estimates from the stochastic volatility model for the Nigerian naira (NGN)

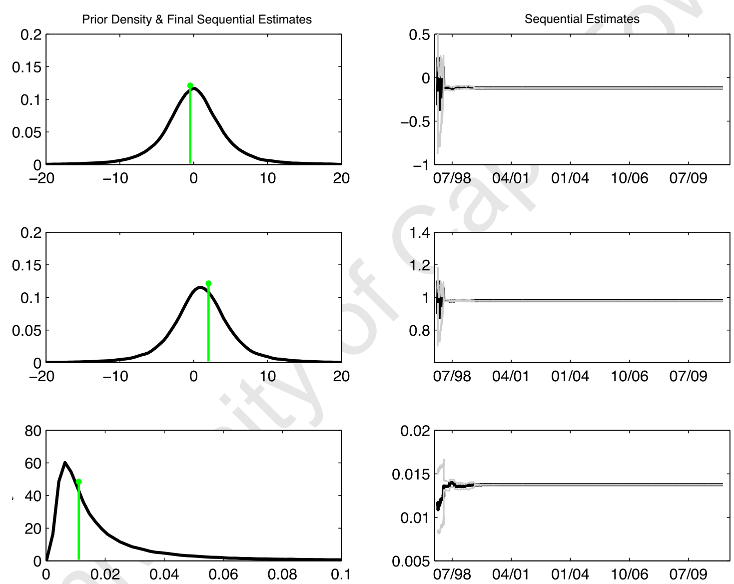


Figure E.40: Parameter estimates from the stochastic volatility model for the Egyptian pound

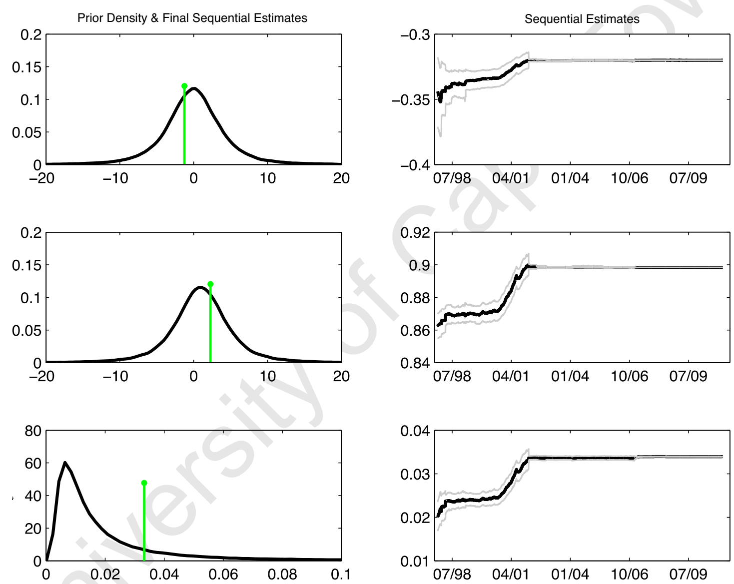


Figure E.41: Parameter estimates from the stochastic volatility model for the Kenyan shilling